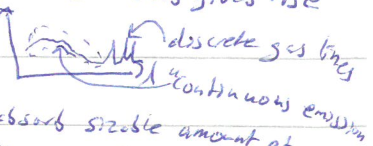
Session 4

The two most common pumping techniques are:

Optical pumping by lasers: (often times a diode laser as they usually have high electrical to optical conversion efficiency > 60% has been demonstrated) overlapping absorption line in the ~~gain~~ medium - i.e. resonant pumping.

by Lamps: When operated at low currents, the emission spectra consist of broadened (because of high pressure) lines characteristic of the gas contained in the bulb, at higher currents more electrons and ions are generated in the gas which can recombine and collide - this gives rise to a continuous emission that will dominate at high currents

Typically not suited for narrow lines in gases



Broadening in solids & liquids ⇒ pump bands instead of sharp levels - can absorb sizable amount of broadband light from lamps! (absorption lines for rare-earths - ex. Yb, Er, Nd tend to not change so much in crystal hosts as they result from transitions between inner "shells" which are shielded by outer electrons - which are more affected by their surroundings. They tend to change more in glasses though!). Lamps radiate in all directions ⇒ need reflective enclosure to ensure efficient pumping - ex. Fig 6.1-6.3

electrical pumping ex. in semiconductor lasers and gas lasers (by discharge), non-resonant electrons excited above desired energy level ~~[illegible]~~ contributes to the heat load!

\# electrons pumped per unit time / unit volume

Lamp pump rate: $R_p = \eta_p \dfrac{P_p}{A l \, h\nu_{mp}}$ ← eq. 6.2.6

P_p ← electric power

assumes uniform pumping! (no spatial dependence)

η_p: pump efficiency; $A l$: pump volume; $h\nu_{mp}$: minimal energy to reach upper laser level directly from ground state

$$\eta_p = \frac{P_r}{P_p} \cdot \frac{P_t}{P_r} \cdot \frac{P_a}{P_t} \cdot \frac{P_{m}}{P_a} = \eta_r \eta_t \eta_a \eta_{pq} \quad \leftarrow \text{eq. } 6.2.5$$

P_r: power in relevant wavelength range; P_p: ~~[illegible]~~ electric power; $\frac{P_r}{P_p} = \eta_r$; $\frac{P_t}{P_r} = \eta_t$; $\frac{P_a}{P_t} = \eta_a$; $\frac{P_m}{P_a} = \eta_{pq}$

P_m: minimum pump power - related to $h\nu_{mp}$

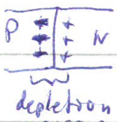
P_a: power absorbed by the ~~medium~~ medium

P_t: power transmitted to the medium by the pump system

Simple picture of a laser diode: make a PN-junction

P | N when in contact holes migrate from P to N and electrons from N to P which leaves charges behind

excess of holes; excess of electrons



⇒ the migrated electrons/holes come into contact with holes/electrons in the P/N side which leads to recombination and leaves a region without carriers = depletion region which gives rise to an electric field inhibiting further carrier diffusion.

A forward bias (P+ & N−) reduces the opposing electric field and holes and electrons flow into the depletion region where they can recombine ~~[illegible]~~ within a certain lifetime - which gives rise to spontaneous emission, using feedback makes it possible to induce stimulated emission before recombination (spontaneous) of all the carriers ⇒ laser.

To scale the power, several junctions can be put into rows and/or columns ⇒ stacks. The dimensions of the junctions are usually different in width and height ⇒ different diffraction angles and get elliptical beams - these can be converted to circular using cylindrical lenses, prisms and/or fibers - Fig. 6.12 & 6.13 & ex. 6.3

Beams from individual junctions can be diffraction-limited but the resulting stacked beam will not be.

Light from individual junctions can be narrow (~1 nm) but temperature gradients and compositional variations between different junctions ⇒ broader linewidth in combined beam

Emission wavelength can be tuned by current and temperature

Low power diodes are usually thermoelectrically cooled while high power diodes tend to be cooled by circulating liquids

Longitudinal pumping = along resonator axis, launched through cavity mirrors ex. Fig 6.11

pump rates $R_p = \frac{\alpha I_p}{h\nu_p}$ ← eq. 6.3.2

α – absorption coefficient $[m^{-1}]$

$h\nu_p$ – pump photon energy

αI_p – absorbed power/volume

(loose focusing) and/or short gain medium

assuming constant pump beam spotsize (W_p) (gaussian beam) and constant laser spotsize (w_0) (gaussian beam, beam waist) in laser rod and assuming that the beam waist is inside the crystal and $R \to \infty$ ("plane wave" or at focus, $\phi = 0$)

gives: $\langle R_p \rangle = \underbrace{\eta_a \eta_r \eta_t}_{= \eta_p} \cdot \frac{P_p}{h\nu_p} \cdot \frac{2}{\pi(w_0^2 + w_p^2)L}$ ← eq. 6.3.12 , $\eta_a \approx 1 - e^{-\alpha L}$

$\langle R_p \rangle$: spatially averaged since gaussians have spatial profiles

increases with smaller w_p but making w_p much smaller means that diffraction has to be considered and w_p might get bigger than w_0 inside of the crystal which wastes gain! ⇒ usually $w_p \approx w_0$

Ideal 4-level systems: (level 1 always empty ⇒ no absorption of laser light!)

3 —
2 —
1 —
0 —

$\langle R_p \rangle_c = \frac{\langle N_2 \rangle_c}{\tau}$ ← eq. 6.3.18 ← pump rate = spontaneous emission rate

c: critical = at threshold

$\langle N_2 \rangle_c = \frac{\gamma}{\sigma_e l}$ ← "gain = losses" eq. 6.3.16 $\gamma = -[\ln(R_1 R_2) + \ln(1 - L_i)]$ ← eq. 1.2.4

⇒ eq. 6.3.20: $P_{th} = \frac{\gamma}{\eta_p} \frac{h\nu_p}{\tau} \frac{\pi(w_0^2 + w_p^2)}{2\sigma_e}$ (1)

Quasi 3-level systems have some population in level 1 ⇒ have to take the absorption of laser light into account

⇒ $[\sigma_e \langle N_2 \rangle_c - \sigma_a \langle N_1 \rangle_c] l = \gamma$ ← eq. 6.3.23 ("gain = losses") , $\langle N_2 \rangle + \langle N_1 \rangle = N_t$ ← doping concentration

⇒ $\langle R_p \rangle_c = \frac{\sigma_a N_t l + \gamma}{(\sigma_a + \sigma_e) l \tau}$ ← eq. 6.3.24 ⇒ $P_{th} = \frac{\sigma_a N_t l + \gamma}{\eta_p} \cdot \frac{h\nu_p}{\tau} \frac{\pi(w_0^2 + w_p^2)}{2(\sigma_e + \sigma_a)}$ (2) ← eq. 6.3.25

from (1) & (2): low threshold power if: • long τ • long pump wavelength (low ν_p) • low losses • high σ_e • small w_0 & w_p – low thresholds in fiber lasers since these can be kept small!

Transverse pumping = ⊥ to resonator axis

can achieve more uniform pumping

eq. 6.3.21 / 6.3.27 (transverse pump) & eq. 6.3.22 / 6.3.26 (lamp pump) assumes cladded laser rod with doping confined to small region outside of which there is no doping

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chap. 6.3.5 compares diode & lamp pumping:

diode pumping is more efficient because of increased pump quantum efficiency (η_{pq}) – since the pumping is resonant, all the electrons are excited to the desired level – which also alleviates thermal load

also state that longitudinal pumping has slightly better absorption than transverse pumping

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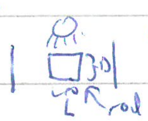
6.4

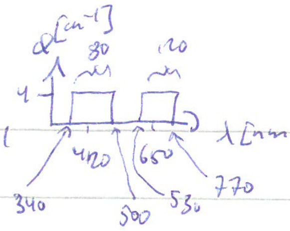


uniform pump, $D = 6\,mm$, $L = 2.5\,cm$, 1% Nd, $\lambda_{mp} = 940\,nm$, $P_{th} = 2\,kW$, $\eta_p = 45\%$, R_p? R_{pc} at threshold

uniform pump ⇒ eq. 6.2.6: $R_p = \eta_p \frac{P_p}{A L h\nu_{mp}} = \left[A = \pi\left(\frac{D}{2}\right)^2, \nu_{mp} = \frac{c}{\lambda_{mp}}\right] = $

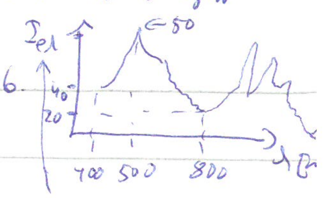
$= 2.01 \cdot 10^{26}\ Hz/m^3$

6.2 $R = 6\,mm$



absorption efficiency

$$\eta_a = \frac{\int (1 - e^{-2\alpha R}) I_{e\lambda}\, d\lambda}{\int I_{e\lambda}\, d\lambda}$$ ← spectral intensity of pump light, Fig 6.6.



called lamp irradiance $[10^{-4}\, W/(cm^2 \cdot nm)]$

2 linear approximations of $I_{e\lambda}$: $I_{e\lambda} = \begin{cases} \frac{50-40}{500-400}\lambda + 50 - \frac{50-40}{500-400}\cdot 500, & 400 \le \lambda \le 500 \\ \frac{20-50}{800-500}\lambda + 20 - \frac{20-50}{800-500}\cdot 800, & 500 \le \lambda \le 800 \end{cases}$

$y = kx + m$, $k = \frac{y_2 - y_1}{x_2 - x_1}$, $y_2 = kx_2 + m \Rightarrow m = y_2 - kx_2$

$\Rightarrow y = \frac{y_2 - y_1}{x_2 - x_1} x + y_2 - \frac{y_2 - y_1}{x_2 - x_1} x_2$ $\Rightarrow I_{e\lambda} = \begin{cases} 0.1\lambda, & 400 \le \lambda \le 500 \\ -0.1\lambda + 100, & 500 \le \lambda \le 800 \end{cases}$ $[10^{-4}\, W/(cm^2 \cdot nm)]$

770 ← interval for non zero values of α

$$\Rightarrow \eta_a = \frac{\int_{400}^{500} (1 - e^{-2\cdot 4 \cdot 0.6}) 0.1\lambda\, d\lambda + \int_{530}^{770} (1 - e^{-2\cdot 4 \cdot 0.6})(-0.1\lambda + 100)\, d\lambda}{\int_{400}^{500} 0.1\lambda\, d\lambda + \int_{500}^{800} (-0.1\lambda + 100)\, d\lambda} = \frac{(1 - e^{-4.8})\left(\left[\frac{0.1}{2}\lambda^2\right]_{400}^{500} + \left[-\frac{0.1}{2}\lambda^2 + 100\lambda\right]_{530}^{770}\right)}{\left[\frac{0.1}{2}\lambda^2\right]_{400}^{500} + \left[-\frac{0.1}{2}\lambda^2 + 100\lambda\right]_{500}^{800}}$$

shortest considered wavelength

← overall the entire lamp spectrum!

$= 0.994$

6.8 $L = 1\,cm$, $\lambda_p = 514.5\,nm$, $\alpha_p = 2\,cm^{-1}$, $\eta_t = 0.95$, $\lambda_{mp} = 616\,nm$, $w_p = 50\,\mu m$, $w_0 = w_p$, $\gamma = 5\%$

η_p & P_{th}? $Ti^{3+}: Al_2O_3$ ← 4 level

longitudinally pumped

6.9 $L = 2\,mm$, $N_t = 3.2 \cdot 10^{20}\,cm^{-3}$, $\lambda_p = 803\,nm$, $w_p = w_0 = 35\,\mu m$, $\eta_t = 80\%$, $\alpha_p = 9\,cm^{-1}$, $\sigma_e = 4.1 \cdot 10^{-20}\,cm^{-2}$

$\tau = 290\,\mu s$, $\gamma = 0.35\%$, P_{th} + explain difference to 6.8? Nd: glass ← 4-level

4-level longitudinally pumped, use eq. 6.3.20: $P_{th} = \frac{\gamma}{\eta_p} \frac{h\nu_p}{\tau} \frac{\pi(w_0^2 + w_p^2)}{2\sigma_e}$

eq. 6.2.8: $\eta_p = \eta_r \eta_t \eta_a \eta_{pq}$, $\eta_{pq} = \left(\frac{h\frac{c}{\lambda_p}}{h\frac{c}{\lambda_{mp}}}\right)^{-1} = \frac{\lambda_p}{\lambda_{mp}}$, $\eta_a = 1 - e^{-\alpha L}$

in 6.8 $\eta_r = 1$ (we don't have any information about the pumping of the pump laser so we only consider its own output)

in 6.9 $\eta_r = 0.5$ (table 6.3 for longitudinally pumping laser diode)

in 6.9 $\eta_{pq} = 0.59$ (table 6.1), in 6.8 $\sigma_e = 4 \cdot 10^{-19}\,cm^2$, $\tau = 3.9 \cdot 10^{-6}\,s$ (table 2.2)

middle step omitting units!

$\Rightarrow \mathrm{I} = \frac{\gamma}{\eta_p} = \begin{cases} 0.0729, & 6.8 \\ 0.0178, & 6.9 \end{cases}$ ($\eta_p = 0.6861$, 6.8)

$\mathrm{II} = \frac{h\nu_p}{\tau} = \begin{cases} 9.3593 \cdot 10^{-14}, & 6.8 \\ 8.0646 \cdot 10^{-16}, & 6.9 \end{cases}$

$\mathrm{III} = \frac{\pi(w_0^2 + w_p^2)}{2\sigma_e} = \begin{cases} 1.9635 \cdot 10^{14}, & 6.8 \\ 9.3865 \cdot 10^{14}, & 6.9 \end{cases}$

$P_{th} = \mathrm{I} \cdot \mathrm{II} \cdot \mathrm{III} = \begin{cases} 1.34\,W, & 6.8 \\ 0.013\,W, & 6.9 \end{cases}$ $\mathrm{I} \cdot \mathrm{III} \sim \begin{cases} 1.43 \cdot 10^{13}, & 6.8 \\ 1.67 \cdot 10^{13}, & 6.9 \end{cases}$ comparable

the big difference mainly come from II because of the longer lifetime in 6.9!

6.10 Yb: YAG (3-level), $L = 1.5\,mm$, 6.5% atomic Yb doping (table 6.2 $\Rightarrow N_t = 9 \cdot 10^{20}\,cm^{-3}$), longitudinal pumped

$\lambda_p = 940\,nm$, $w_p = w_0 = 45\,\mu m$, $\lambda_s = 1.03\,\mu m$, $\sigma_e = 1.9 \cdot 10^{-20}\,cm^{-3}$, $\sigma_a = 0.11 \cdot 10^{-20}\,cm^{-3}$, $\tau = 1.5\,ms$, $\gamma = 2\%$

P_{th}?

3-level longitudinal pump $\Rightarrow$ eq. 6.3.25: $P_{th} = \frac{\sigma_a N_t L + \gamma}{\eta_p} \cdot \frac{h\nu_p}{\tau} \frac{\pi(w_0^2 + w_p^2)}{2(\sigma_e + \sigma_a)} = \left[\nu_p = \frac{c}{\lambda_p} \rightarrow\right.$

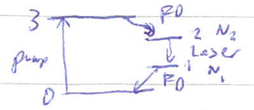
↳ assume $\eta_p \approx \eta_a = 1 - e^{-\alpha L}$ - no info about pump optics$\left.\right] \approx 140\,mW$

$\alpha = 5\,cm^{-1}$ table 6.2

Note extra terms because of reabsorption ($\sigma_a N_t L$ & σ_a) compared to 4 level eq.

rate eqs. assume balance between total number of atoms undergoing a transition & #photons created/annihilated

4-level (need $\tau_1 \ll \tau_{21}$ to maintain inversion; here we consider $\tau_1 \ll \tau_{21}$ so that $N_1 \approx 0$)



FD = fast decay

No population in 3 or 1!

eq. 7.2.1:

$$\frac{dN_2}{dt} = R_p - BN_2\phi - \frac{N_2}{\tau}$$

pump rate; stimulated emission; ← spontaneous emission + non-radiative decay

$$\frac{d\phi}{dt} = V_a B N_2 \phi - \frac{\phi}{\tau_c}$$

← loss term; τ_c ← life time in cavity; ϕ = # photons

V_a: mode volume in active medium; #photons/time/volume from stimulated emission

define logarithmic losses γ similar to chap. 1 (eq. 7.25-7.27)

the gain & loss for a round trip will then be given by:

$e^{-2\gamma + 2\sigma N_2 l}$ (l ← length of active medium) eq. 7.2.9, assume $\sigma N_2 l - \gamma \ll 1$ (comparable loss & gain)

using $I \propto \phi \Rightarrow B = \frac{\sigma c}{V}$ ← eq. 7.2.13 (V mode volume within laser cavity)

$\tau_c = \frac{L_e}{\gamma c}$ ← eq. 7.2.14 (L_e optical length of resonator (account for refractive index))

inversion: $N_2 - N_1 \approx N_2 \Rightarrow$ can exchange N_2 for N in eq. 7.2.1 → eq. 7.2.16

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critical inversion when $V_a B N_c - \frac{1}{\tau_c} = 0$ (gives $\frac{d\phi}{dt} = 0$ RHS in eq. 7.2.1)

$\Rightarrow N_c = \frac{1}{V_a B \tau_c} = \frac{\gamma}{\sigma l}$ ← eq. 7.3.2, $\frac{dN}{dt} = 0$ & $\phi = 0 \Rightarrow R_{cp} = \frac{N_c}{\tau} = \frac{\gamma}{\sigma l \tau}$ ← eq. 7.3.3 (critical pump rate)

Note that R_{cp} is higher for quasi 3-level!

CW: $\frac{dN}{dt} = \frac{d\phi}{dt} = 0 \Rightarrow N_0 = \frac{\gamma}{\sigma l} = N_c$ ← eq. 7.3.4

$\phi_0 = V_a \tau_c \left[R_p - \frac{N_0}{\tau}\right]$ ($\frac{N_0}{\tau}$: stored energy)

R_p increases inversion below threshold and increases #photons above!

uniform pumping (chap. 6) into eq. for $R_{cp} \Rightarrow P_{th} = \frac{\gamma}{\eta_p}\frac{h\nu_{mp}}{\tau}\frac{A}{\sigma}$ ← eq. 7.3.12 (P_{th}: threshold pump power; A ← area of active medium)

$\Rightarrow \phi_0 = \frac{A_b \gamma}{\sigma}\frac{\tau_c}{\tau}(x-1)$, $x = \frac{R_p}{R_{cp}} = \frac{P_p}{P_{th}}$ ← eq. 7.3.8 (A_b: cross-sectional mode area)

$$\boxed{\frac{1}{\tau_c} = \frac{c}{L_e}\overbrace{(\gamma_i + \gamma_1 + \gamma_2)}^{\gamma}}$$

internal losses; mirror 1 losses; mirror 2 losses

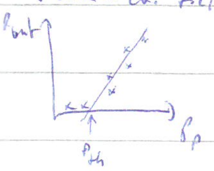
slope efficiency

$\eta_s = \frac{dP_{out}}{dP_p} = \eta_p \frac{\gamma_2}{2\gamma}\frac{h\nu}{h\nu_{mp}}\frac{A_b}{A}$ ← eq. 7.3.13

$P_{out} = A_b \frac{h\nu}{\sigma\tau}\frac{\gamma_2}{2}\left[\frac{P_p}{P_{th}} - 1\right]$ ← eq. 7.3.9 ($I_s = \frac{h\nu}{\sigma\tau}$ = saturation intensity)

output rate/loss rate through mirror 2: $\frac{c\gamma_2}{L_e} \Rightarrow P_{out} = \frac{\gamma_2 c}{2L_e} h\nu \phi$ ← # photons in cavity; $h\nu$ ← photon energy; per unit time (if mirror 1 has R=1) eq. 7.2.18

linear, can estimate P_{th} by linear fit of data - ex. 7.2.23



ex. 7.1 using eq. 7.2.18: $\phi = \frac{P_{out} \cdot L_e}{-\ln(1-T)c \cdot h\frac{c}{\lambda}}$ (ϕ: # photons in cavity)

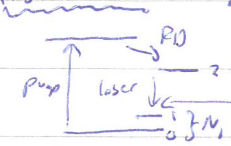
HeNe: $L_e = 50$ cm, $\lambda = 630$ nm, $P_{out} = 10$ mW, $T = 1\% \Rightarrow \phi \sim 10^{10}$

CO_2: $L_e = 150$ cm, $\lambda = 10.6\,\mu m$, $P_{out} = 10$ kW, $T = 45\% \Rightarrow \phi \sim 10^{16}$

$\Rightarrow$ neglect "extra" starting photon in eqs. for $\frac{d\phi}{dt}$ - see eq. 7.2.2 (from spontaneous emission)

Quasi 3-level

$\Delta E_{01} \sim kT \Rightarrow$ non-negligible population in N_1 - absorption of laser photons - extra losses!



sublevel of ground state

$$\frac{dN_2}{dt} = R_p - (B_e N_2 - B_a N_1)\phi - \frac{N_2}{\tau}$$

(stimulated emission; absorption of laser light from N_1!)

$$\frac{d\phi}{dt} = V_a(B_e N_2 - B_a N_1)\phi - \frac{\phi}{\tau_c}$$ ← 7.2.14

$B_e = \frac{\sigma_e c}{V}$ $B_a = \frac{\sigma_a c}{V}$ ← eq. 7.2.20

use that $N_1 + N_2 = N_t$ ← total doping conc.

define $f = \frac{\sigma_a}{\sigma_e}$ & $N = (N_2 - fN_1)$ ← inversion

$\Rightarrow \frac{dN}{dt} = R_p(1+f) - \frac{(\sigma_e+\sigma_a)c}{V}N\phi - \frac{fN_t + N}{\tau_c}$

$\frac{d\phi}{dt} = \left(\frac{V_a \sigma_e c}{V}N - \frac{1}{\tau_c}\right)\phi$ ← eq. 7.2.24

R_p considered constant - the more we approach a pure 3 level system, the more R_p will be altered by N_1 and not be constant!

$\frac{d\phi}{dt} = 0 \Rightarrow N_c = \frac{\gamma}{\sigma_e l}$ ← eq. 7.4.1 like 4-level

$\frac{dN}{dt} = 0, \phi = 0 \Rightarrow R_{cp} = \frac{fN_t + N_c}{(1+f)\tau}$ ← eq. 7.4.2

uniform pump:

$P_{th} = \frac{\gamma\left(1 + \frac{\sigma_a N_t l}{\gamma}\right)}{\eta_p}\frac{h\nu_p}{\tau}\frac{A}{\sigma_e + \sigma_a}$ ← eq. 7.4.4

$\frac{dN}{dt} = \frac{d\phi}{dt} = 0 \Rightarrow N_0 = N_c$

$\phi_0 = \frac{V}{N_0(\sigma_e + \sigma_a)c} \cdot \frac{fN_t - N_0}{\tau}(x-1)$ ← eq. 7.4.6

$x = \frac{R_p}{R_{cp}} = \frac{P_p}{P_{th}}$

$P_{out} = \frac{A_b(1+B)}{\eta_p}\frac{h\nu_p}{\tau}\left(\frac{A}{\sigma_e + \sigma_a}\right)$

$B = \frac{\sigma_a N_t l}{\gamma}$ ← eq. 7.4.4

$\eta_s = \eta_{s\,4\,level}$ ← eq. 7.4.10

because absorbed laser photons raised electrons to upper level where they are available for stimulated emission!

This analysis has assumed uniform pumping & mode energy density – space independent model → is and neglects standing wave character, which is highly idealized but is decent if the laser has many longitudinal modes (the standing waves will start canceling each other out), if the laser is also operating in multimode transversely – the ~~resulting~~ beating beam profile will also be quite uniform

(x) the beam radius $w(z)$ are assumed to be constant and no standing wave effects are taken into account

They also consider uniform pump & Gaussian mode

chap. 7.3.2 & 7.4.2 consider Gaussian(x) pump & mode and find that the expressions for N_c, N_0 and R_{cp} remain the same but will now be true for the spatially averaged quantities $\langle N_c \rangle$, $\langle N_0 \rangle$, $\langle R_p \rangle_c$

see eqs. 7.3.19, 7.3.21 & 7.4.12, 7.4.13 (set $f = \frac{\sigma_a}{\sigma_e}$ in this eq. to see that it's the same as 7.4.2)

- it is found that the slope efficiency increases with increased pump(xx) power – as there will be more photons in the cavity and hence more photons at the "edges" of the mode which increases the amount of stimulated emission in the lower intensity parts of the mode. This increase is slower for a Gaussian pump than a uniform one as it has less intensity in its wings
- Accounting for standing wave effects also shows that the slope efficiency increases with the pump power – as stimulated emission can happen more efficiently further away from the standing waves' peaks

(xx) this increase is slower for 3 level because of reabsorption in the wings, this difference is not so pronounced if the beam spot is much smaller than the pump beam

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chap. 7.5: Maximizing output power – increasing mirror transmission leads to more outcoupling but also reduces cavity photons which can reduce the output power! → need to balance!

for 4 level space independent

eq. 7.3.9: $P_{out} = A_b I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)$, $P_{th} = \frac{\gamma}{\eta_p}\frac{h\nu_{mp}}{\tau}\frac{A}{\sigma}$ → eq. 7.3.16

$G = \left[\gamma = \gamma_1 + \frac{1}{2}(\gamma_i + \gamma_2) = (\gamma_1 + \gamma_{1/2}) \cdot \frac{\gamma_1 + \frac{1}{2}(\gamma_i + \gamma_2)}{\gamma_1 + \gamma_{1/2}}\right] = \underbrace{\frac{h\nu_{mp} A}{\eta_p \tau \sigma}(\gamma_1 + \gamma_{1/2})}_{P_{mth}} \cdot \frac{\gamma_1 + \frac{1}{2}(\gamma_{i} + \gamma_2)}{\gamma_1 + \gamma_{1/2}}$ ← eq. 7.5.1

no losses at mirror 2

insert 7.5.1 into 7.3.9 & define $S = \frac{\gamma_2/2}{\gamma_1 + \gamma_{1/2}} \propto \gamma_2$ → $\frac{dP_{out}}{dS} = 0$ ⇒ $S_{op} = \sqrt{x_m} - 1$ ← eq. 7.5.5 (optimal)

eq. 7.5.3 →, eq. 7.5.4 → $x_m = \frac{P_p}{P_{mth}}$

$P_{op} = A_b I_s \left(\gamma_1 + \frac{\gamma_i}{2}\right)\left(\sqrt{x_m} - 1\right)^2$ ← eq. 7.5.6

optimum reflectivity/transmission for given pump!

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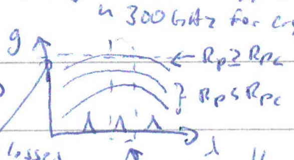
about modes $L = 1\,m$ ⇒ $\Delta\nu = \frac{c}{2L} = 150\,MHz$, gain bandwidth ~1 GHz Doppler broadening in gas, ~300 GHz for crystal ion in solid state material

⇒ thousands of modes within gain!

Homogeneous broadening – gain maintains its shape ⇒

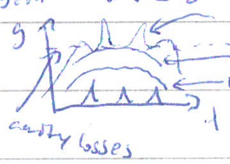


(spectral hole burning)

↑ more gain than other 2 ⇒ only this one will resonate*

Inhomogeneous broadening – gain can be* depleted at different wavelengths

1 mode resonates at $R_p = R_{pc}$ but more modes will resonate as $R_p > R_{pc}$ when that part of the gain curve matches the losses



(spatial hole burning)

Standing waves – deplete inversion where* there are peaks ⇒ other modes can deplete inversion at the nodes for other modes when pumped above threshold ⇒ homogeneously broadened materials can also operate with multi longitudinal modes!

spatial hole burning is not so efficient for inhomogeneously broadened materials as different modes (wavelengths) will interact with different ions ⇒ hole from one mode not so useful for other mode

- Multi longitudinal modes in hom. broadened material smears out gain along the medium ⇒ only modes close to gain peak will resonate – i.e. fewer modes than for inhom. broadening!

– no σ_e for $\lambda = 1.053\,\mu m$

7.5 From text: $\lambda = 1.047\,\mu m$, $\sigma_e = 1.8\cdot 10^{-19}\,cm^2$, $\tau = 480\,\mu s$, $\gamma_i \approx \eta_p$ the same as in ex. 7.2

From ex. 7.2! $R_1 = 100\%$, $L = 7.5\,cm$, $D = 6.35\,mm$, $R_2 = 85\%$, $\eta_p = 8.5\%$; $\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$ (we keep the same mirrors) $\gamma = 0.12$, $\gamma_2 = -\ln R_2 = 0.162$; $\lambda_{mp} = 0.44\,\mu m$

$A_b = 0.23\,cm^2$ $\gamma_i = 0.038$

↑ mode area in active medium

↓ multi long. + transverse modes ⇒ spat. indep. model!

$N_c^{YLF} = \frac{\gamma}{\sigma_e L} \approx 8.9\cdot 10^{16}\,cm^{-3}$ eq. 7.3.3 changing Nd:YAG ⟶ Nd:YLF (both 4 level) only changing to th material!

($N_c^{YAG} \approx 5.7\cdot 10^{16}\,cm^{-3}$ – $\sigma_e = 2.8\cdot 10^{-19}\,cm^2$ ⇒ need less inversion!)

$P_{th}^{YLF} = \frac{\gamma}{\eta_p}\frac{h\nu_{mp}}{\tau}\left(\frac{A}{\sigma_e}\right) = \left[\nu_{mp} = \frac{c}{\lambda_{mp}}\ h = 6.626\cdot 10^{-34}\,Js\right] \approx 1.69\,kW$ ← eq. 7.3.12 ($P_{th}^{YAG} \approx 2.26\,kW$ $\tau^{YAG} = 230\,\mu s$)

$\eta_s^{YLF} = \frac{dP_{out}}{dP_p} = \frac{A_b h\nu}{\sigma_e \tau}\frac{\gamma_2}{2}\frac{1}{P_{th}} = \left[\nu = \frac{c}{\lambda}\right] \approx 0.024 = 2.4\% = \eta_s^{YAG}$

↑ eq. 7.3.11

$\sigma_e^{YAG} = 2.8\cdot 10^{-19}\,cm^2$

biggest difference compared which σ_e^{YLF} & τ^{YLF} has the most impact!

(eq. 7.3.12 in eq. 7.3.11 ⇒ $\eta_s = \frac{A_b}{A}\cdot\frac{\nu}{\nu_{mp}}\cdot\frac{\gamma_2}{\gamma}\cdot\frac{\eta_p}{2}$ – all the same on these cases except for ν since $\lambda^{YLF} = 1.047\,\mu m$, $\lambda^{YAG} = 1.06\,\mu m$ fractions with ν_{mp} are similar)

7.6 $P_p = 7\,kW$, T_2^{opt} & P_{out}^{opt}?

eq. 7.5.5 $S_{op} = \sqrt{x_m} - 1 = \left[x_m = \frac{P_p}{P_{mth}}\right.$ ← eq. 7.5.4 $P_{mth} = P_{th}\cdot\frac{\gamma_i + \gamma_1/2}{\gamma}$ ← eq. 7.5.1 $= \left[\gamma_1 = 0,\ \gamma_i = 0.038\ \ \gamma = 0.12\ \ P_{th} = P_{th}^{YLF} = 1.69\,kW\right]$

⇒ $x_m = \frac{7}{1.69\cdot\frac{0.038}{0.12}} \approx 13\Big] \approx 2.6$, $S = \frac{\gamma_2/2}{\gamma_i + \gamma_1/2}$ ⇒ $[\gamma_1 = 0]$ ⇒ $\gamma_2 = 2S_{op}\gamma_i$

eq. 7.5.3

$\gamma_2 = -\ln R_2$ ⇒ $T_2 = 1 - R_2 = 1 - e^{-\gamma_2} = 1 - e^{-2\cdot 2.6\cdot 0.038} \approx 18\%$ ($R_2 \approx 82\%$ a bit lower than 85% in ex. 7.2)

$P_{op} = A_b\cdot\underbrace{\frac{h\nu}{\sigma\tau}}_{I_s}\left(\gamma_i + \frac{\gamma_1}{2}\right)\left[\sqrt{x_m} - 1\right]^2 = \left[\gamma_1 = 0\ \ x_m = 13\ \ \sigma = \sigma_e = 1.8\cdot 10^{-19}\,cm^2\ \ A_b = 0.23\,cm^2\ \ \nu = \frac{c}{\lambda},\ \lambda = 1.047\,\mu m\ \ \tau = 480\,\mu s\ \ \gamma_i = 0.038\right]$

eq. 7.5.6

$\approx 130\,W$ (compared to about 120 W at the same pump power for Nd:YAG ($x_m \approx 10$) see ex. 7.6)

adapted version of example 7.4

$R_1 = 100\%$ $R_2 = 95\%$

internal + mirror losses ⇒ $\gamma = 0.03$ $\sigma_e = 2.8\cdot 10^{-19}\,cm^2$

gaussian pump ≈ mode with $w_0 = w_p = 130\,\mu m$ ⇒ spatially dependent! $\gamma_2 = 0.05$ $\tau = 230\,\mu s$

use eq. 7.3.34 : $x = \frac{y}{1 - \frac{\ln(1+y)}{y}}$ $x = \frac{P_p}{P_{mth}}$ $P_{mth} = \frac{\gamma}{\eta_p}\frac{h\nu_p}{\tau}\frac{\pi w_p^2}{2\sigma_e} \approx 38\,mW$ $\eta_p = 81\%$

$\lambda_p = 807\,nm$

eq. 7.3.32 $P_p = 1.14\,W$

$P_{mth} = 38\,mW$ ⇒ $x = \frac{1.14}{0.038} = 30$ $y = \frac{P_{out}}{P_s}$ ← eq. 7.3.26 ← eq. 7.3.27 $\lambda = 1.06\,\mu m$

$P_s = \frac{\gamma_2}{2}\frac{\pi w_0^2}{2}\cdot\underbrace{\frac{h\nu}{\sigma_e\tau}}_{I_s}$ ← eq. 7.3.28

guess values for y to set RHS of eq. 7.3.24 equal to $x = 30$!

ex. $y = 30$ ⇒ $x = \frac{30}{1 - \frac{\ln(1+30)}{30}} \approx 33$, $y = 20$ ⇒ $x \approx 24$ ... $y = 26$ ⇒ $x = 30$

$y = 26$ ⇒ $P_{out} = 26 P_s = 26\cdot\frac{\gamma_2}{2}\frac{\pi w_0^2}{2}\frac{h\nu}{\sigma_e\tau} \approx 500\,mW$

7.17 given: $\lambda = 514.5\,nm$, $\Delta\nu_0^* = 3.5\,GHz$, $L_e = 120\,cm$, $L = 100\,cm$, $\gamma = 10\%$, $\sigma_e = 2.5\cdot 10^{-13}\,cm^2$, $\tau = 5\,ns$, $\tau_1 \ll \tau_2$, one cavity mode overlaps transition peak

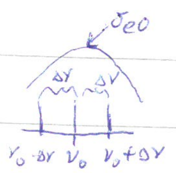
Doppler broadening ⇒ inhomogeneous! Ar 4-level ⇒ $N_c = \frac{\gamma}{\sigma_e L} = 4\cdot 10^{9}\,cm^{-3}$ ← eq. 7.3.2 $R_{cp} = \frac{\gamma}{\sigma_e\tau L} = 8\cdot 10^{17}\,cm^{-3}s^{-1}$

eq. 7.3.3

eq. 2.4.17 & eq. 2.4.18 ⇒ $\sigma \propto W \propto g$ (lineshape) ($W_{12}^{sa} = W_{21}^{sa}$ ← eq. 2.4.12)

$g(\nu - \nu_0) = \underbrace{\frac{2}{\Delta\nu_0^*}\sqrt{\frac{\ln 2}{\pi}}}_{constant}\, e^{-\frac{4(\nu-\nu_0)^2}{\Delta\nu_0^{*2}}\ln 2}$ ← eq 2.4.24 for inhomogeneous line ⇒ $\sigma_e(\nu - \nu_0) = \sigma_{e0}\, e^{-\frac{4(\nu-\nu_0)^2}{\Delta\nu_0^{*2}}\ln 2}$

↖ given at peak!



⇒ $\frac{R_{cp2}}{R_{cp1}} = \frac{\frac{\gamma}{\sigma_e(\Delta\nu)\tau L}}{\frac{\gamma}{\sigma_e(0)\tau L}} = \frac{\sigma_e(0)}{\sigma_e(\Delta\nu)} = e^{\frac{4\Delta\nu^2}{\Delta\nu_0^{*2}}\ln 2} = \left[\Delta\nu = \frac{c}{2L_e} = 0.125\,GHz\right] \approx 1.003$

(0.15 GHz in the book)

using L instead of the gain length   cavity length