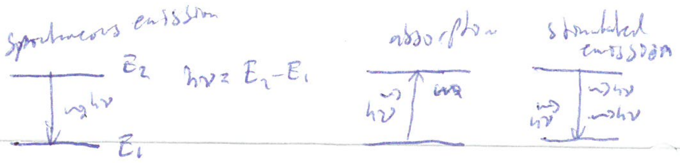
Session 1

1.3

Fig. 1.1



visible range: 400–800 nm ⇒ $\lambda = 600\,nm$, $T = 300\,K$

At thermal equilibrium, the populations of the two levels are described by Boltzmann statistics, eq. 1.2.2: $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{E_2 - E_1}{kT}}$ g_2, g_1 = degeneracy factors = # of levels that have the same energy

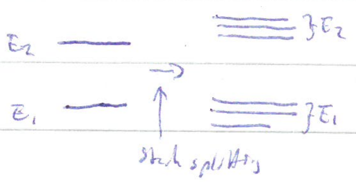
assuming $g_2 = g_1$, using $E_2 - E_1 = h\nu = h\frac{c}{\lambda}$ ← $c = 3\cdot10^{8}\,m/s$, $h = 6.626\cdot10^{-34}\,Js$

gives:

$$\frac{N_2}{N_1} = e^{-\frac{hc}{\lambda kT}} \approx 2\cdot10^{-35}$$

($k = 1.38\cdot10^{-23}\,J/K$)

⇒ essentially all atoms are in the ground state, as $kT \simeq 4\cdot10^{-21}\,J \ll E_2 - E_1 = h\frac{c}{\lambda} \simeq 3\cdot10^{-19}\,J$ the energy from thermal contributions is insufficient to excite the atoms.



However, degenerate levels may acquire slightly different energies from ex. Stark splitting – the energy differences between these sublevels in the manifolds (E_2, E_1) might be small enough to populate higher Stark levels than the lowest one at room temperature!

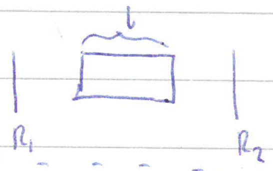
1.4

eq. 1.2.2: $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{E_2 - E_1}{kT}}$ → assume $g_2 = g_1$ & use $E_2 - E_1 = h\nu$ ⇒ $\frac{1}{e} = e^{-\frac{h\nu}{kT}}$ → $\ln e = \ln e^{\frac{h\nu}{kT}}$ →

↳ $1 = \frac{h\nu}{kT}$ → $\nu = \frac{kT}{h} = [k = 1.38\cdot10^{-23}\,J/K,\ T = 300\,K,\ h = 6.626\cdot10^{-34}\,Js] \approx 6.25\cdot10^{12}\,Hz$ ← THz

$\lambda = \frac{c}{\nu} = \frac{3\cdot10^{8}}{6.25\cdot10^{12}} \simeq 48\cdot10^{-6}\,m$ ← μ Far infrared region!

1.5



propagation in laser material: $\underbrace{dF}_{\text{difference in flux}} = (\underbrace{\sigma_{21} N_2 F}_{\text{emission}} - \underbrace{\sigma_{12} N_1 F}_{\text{absorption}})\underbrace{dz}_{\text{length}}$ →

↳ ⇒ $\frac{dF}{dz} = (\sigma_{21} N_2 - \sigma_{12} N_1)F = [\text{eq. 1.1.9: } g_2\sigma_{21} = g_1\sigma_{12}] = \sigma_{21}(N_2 - \frac{g_2}{g_1}N_1)F$

(x)

$F' - f(x)F = 0$ → $\underbrace{F'e^{-\int f(x)dx} - f(x)e^{-\int f(x)dx}F}_{= (Fe^{-\int f(x)dx})'} = 0$

↓

$Fe^{-\int f(x)dx} = C$ → $F = Ce^{\int f(x)dx}$

↳ ⇒ $F = F(0)\, e^{\sigma_{21}(N_2 - \frac{g_2}{g_1}N_1)l}$ (x)

the other ↓ round-trip cavity losses

$F_{RT} = \underbrace{F e^{\sigma(N_2 - \frac{g_2}{g_1}N_1)l}(1-L_i)R_2}_{\text{what remains after propagation through cavity and reflection from } R_2} e^{\sigma(N_2 - \frac{g_2}{g_1}N_1)l}(1-L_i)R_1$

(F: flux in cavity)

At threshold $F_{RT} = F$ ⇒ $1 = e^{2\sigma(N_2 - \frac{g_2}{g_1}N_1)l}(1-L_i)^2 R_1 R_2$

⇒ $2\sigma\underbrace{(N_2 - \frac{g_2}{g_1}N_1)}_{N_c = \text{critical/threshold inversion}}l = \ln\left(\frac{1}{(1-L_i)^2 R_1 R_2}\right)$ → $\begin{cases} N_c = -\dfrac{\ln R_1 + \ln R_2 + 2\ln(1-L_i)}{2\sigma l} & \leftarrow \text{eq. 1.2.3} \\ N_c = \dfrac{\gamma}{\sigma l} & \leftarrow \text{eq. 1.2.5} \end{cases}$

logarithmic losses

$\gamma_1 = -\ln R_1$

$\gamma_2 = -\ln R_2$

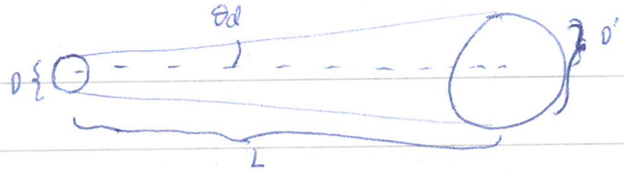
$\gamma_i = -\ln(1-L_i)$

$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$ ← per pass

given: $R_1 = 1$, $R_2 = 0.5$, $L_i = 0.01$, $l = 7.5\,cm$, $\sigma = 28\cdot10^{-19}\,cm^2$

$$\gamma = -\left(\frac{\ln R_1 + \ln R_2}{2} + \ln(1-L_i)\right) \approx 0.357 \Rightarrow N_c = \frac{\gamma}{\sigma l} \simeq 1.7\cdot10^{17}\,cm^{-3}$$

1.6



given: $D = 1m$, $\lambda = 694nm$, perfect spatial coherence
$L = 384\,000\,km$

eq. 1.4.1: $\theta_d = \frac{\beta\lambda}{D}$ ← aperture diameter $\beta = 1$ for perfect spatial coherence
↑ diffraction angle

$\tan\theta_d = \frac{D'/2}{L} \Rightarrow D' = 2\tan\theta_d \cdot L = 2L\tan\frac{\lambda}{D} \approx \frac{2L\lambda}{D} \approx 532m$ ($\frac{\lambda}{D} << 1$)

1.7 eq. 1.4.5 $B = \left(\frac{2}{\beta\pi\lambda}\right)^2 P = [\beta = 1,\ \lambda = 514.5nm] = 1.5\cdot10^{12}\ W/m^2 sr$

For a more thorough discussion about brightness than what's mentioned in the book, see the supplementary material to this task on the course webpage. Note however that the answer is wrong there (it says 240GW/m²sr).

2.3 Planck's law: $\rho_\nu = \frac{8\pi\nu^2}{c_n^3}\frac{h\nu}{e^{h\nu/kT}-1}$ ← eq. 2.2.22 c_n = velocity of light in medium $= \frac{c}{n}$ ← refractive index

↑ spectral density of em. radiation from black body in thermal equilibrium at temperature T

- spectral density means divided by $d\nu$!, what is preserved when going from $\nu \to \lambda$ is:

$\rho_\nu d\nu = \rho_\lambda d\lambda$, $\frac{d\nu}{d\lambda} = \frac{d}{d\lambda}\left(\frac{c}{\lambda}\right) = -\frac{c}{\lambda^2} \Rightarrow d\nu = -\frac{c}{\lambda^2}d\lambda$ we want a positive spectral density, - only indicates inverse scaling of $d\nu$ and $d\lambda$

$\Rightarrow \rho_\lambda d\lambda = \frac{8\pi\left(\frac{c}{\lambda}\right)^2}{\left(\frac{c}{n}\right)^3}\frac{h\left(\frac{c}{\lambda}\right)}{e^{hc/\lambda kT}-1}\cdot\left(+\frac{c}{\lambda^2}\right)d\lambda \Rightarrow \rho_\lambda = \frac{8\pi hcn^3}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} = K\cdot\lambda^{-5}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}$

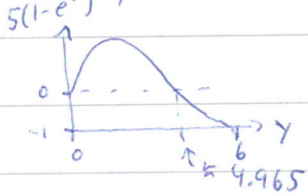
extreme points where $\frac{d\rho_\lambda}{d\lambda} = 0$: $K\left[-5\lambda^{-6}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1} - \lambda^{-5}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-2}e^{\frac{hc}{\lambda kT}}\cdot\left(\frac{-hc}{\lambda^2 kT}\right)\right] = 0 \to$

$\Leftrightarrow 5\left(e^{\frac{hc}{\lambda kT}}-1\right) - \frac{hc}{\lambda kT}e^{\frac{hc}{\lambda kT}} = 0 \to 5\left(1-e^{-\frac{hc}{\lambda kT}}\right) = \frac{hc}{\lambda kT}$ use: $\lambda_m T = \frac{hc}{ky} \to y = \frac{hc}{\lambda kT} \to$

$= \lambda$ ↑ from instructions
Wien's displacement law

$\Leftrightarrow 5(1-e^{-y}) = y$

plot in MATLAB



2.7 Homogeneous broadening: effects that broaden transitions' linewidths by affecting different radiating or absorbing atoms/molecules the same, the lineshape is described by a Lorentz function

all atoms/molecules have the same resonance frequencies

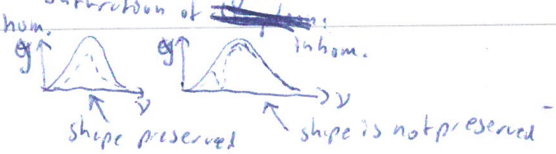
ex. natural linewidth (because of finite lifetime of energy levels)
collisions in gas (all atoms/molecules experience the same average rate of collisions)

Inhomogeneous broadening: effects broadening transition linewidths by making different radiating/absorbing atoms/molecules interact with different wavelengths, the lineshape is Gaussian

different atoms/molecules have different resonance frequencies

ex. different velocities of particles in a gas - Doppler broadening
laser ions experiencing different local fields (electric and/or magnetic) ⇒ different Stark split
↳ ex. in glass (amorphous material)

Saturation of gain:



- section 2.8.2 - 2.8.3

⇘

↘

given: Doppler broadening (inhomogeneous), $\lambda = 1.15\,\mu m$, $\Delta\nu_0^* = 9 \cdot 10^8$ Hz, $\tau_{sp} = 10^{-7}$ s

eq. 2.4.25: $\sigma_{in} = \frac{2\pi^2}{3n\varepsilon_0 ch} |\mu|^2 \nu g_t(\nu - \nu_0)$

$g_t = g^*(\nu-\nu_0) = \frac{2}{\Delta\nu_0^*}\sqrt{\frac{\ln 2}{\pi}}\, e^{-\frac{4(\nu-\nu_0)^2}{\Delta\nu_0^{*2}}\ln 2}$ ← eq. 2.4.27

inhomogeneous

eq. 2.3.15: $\tau_{sp} = \frac{3h\varepsilon_0 c^3}{16\pi^3\nu_0^3 n |\mu|^2}$ $\left(= \frac{1}{A}\right)$ ← Einstein coefficient = rate of spontaneous emission

$\max(g_t) = \frac{2}{\Delta\nu_0^*}\sqrt{\frac{\ln 2}{\pi}}$

$\Rightarrow \max(\sigma_{in}) = \frac{2\pi^2}{3n\varepsilon_0 ch} \cdot \underbrace{\frac{3h\varepsilon_0 c^3}{16\pi^3\nu_0^3 n\tau_{sp}}}_{|\mu|^2 \text{ from } 2.3.15} \cdot \nu_0 \cdot \frac{2}{\Delta\nu_0^*}\sqrt{\frac{\ln 2}{\pi}} = \left[\nu_0 = \frac{c}{\lambda}\right] = \frac{(\lambda/n)^2}{4\pi\tau_{sp}\Delta\nu_0}\cdot\sqrt{\frac{\ln 2}{\pi}} = \rightarrow$

$\nu = \nu_0$ at the peak

$\sigma = [n \approx 1 \text{ in the gas}] \approx 5.5 \cdot 10^{-16}\, m^2 = 5.5 \cdot 10^{-12}\, cm^2$

$|\mu|$ = magnitude of electric dipole moment if $\neq 0$ electric dipole allowed transition — occurs between states whose wavefunction have opposite parity (section 2.3.3), if $|\mu| = 0$ the transition cannot occur as result of an electric-dipole interaction but can for example occur by interaction with a magnetic field (light carries both!), the probability for an electric dipole transition is 10^5 greater than that of a magnetic dipole (section 2.4.2)

Also note that $\tau_{sp} \propto \frac{1}{\nu_0^3} \propto \lambda^3$ which means that the spontaneous emission increases rapidly with shorter wavelengths $\Rightarrow$ difficulties with obtaining population inversion

example 2.1