Unstable optical resonators

Laser Physics course SK3410
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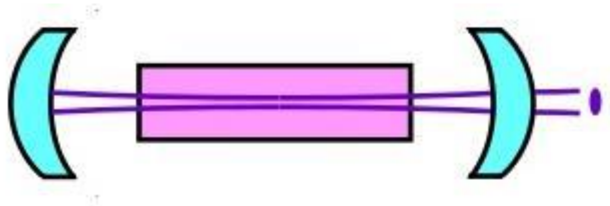
KTH ICT OFO

Outline

- **Types of resonators**
- Geometrical description
- Mode analysis
- Experimental results
- Other designs of unstable resonators
- Conclusions

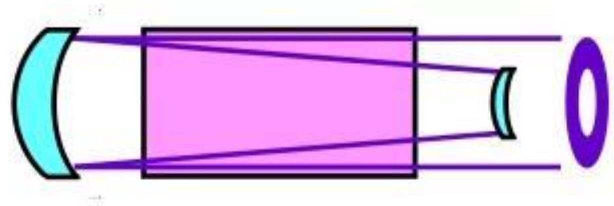
Types of resonators

Stable



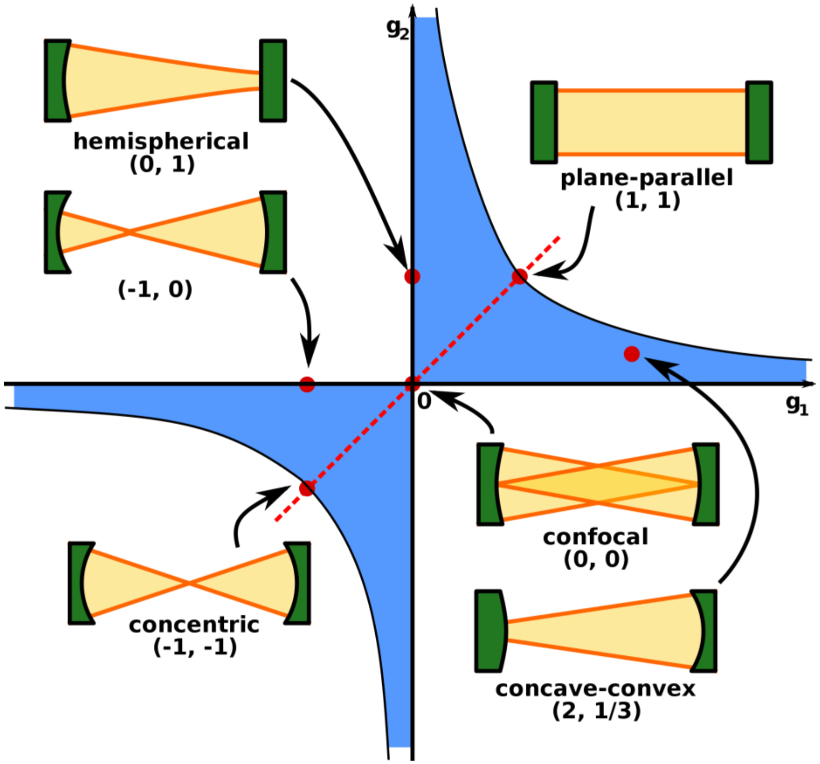
Beam is maintained inside limited volume of active medium

Unstable



Beam expands more and more with each bounce

Resonator stability



$$g_1 = 1 - \frac{L}{R_1}, \qquad g_2 = 1 - \frac{L}{R_2}$$

Stability condition :

$$0 \leqslant \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leqslant 1.$$

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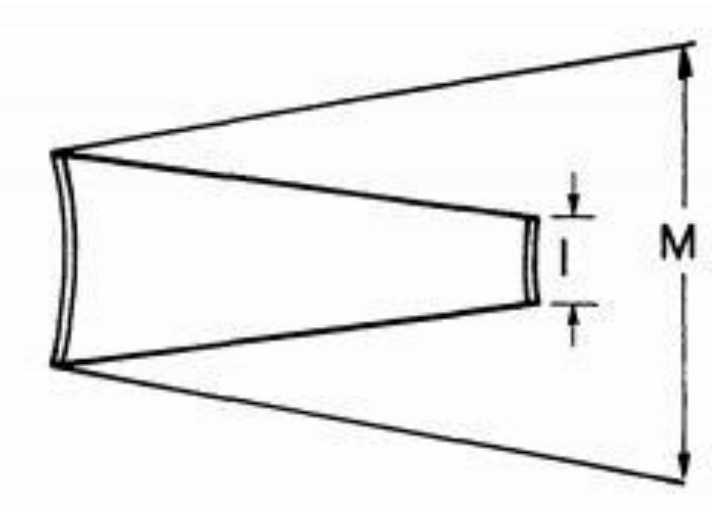
Geometrical description



Magnification per roundtrip :

$$M = I_{out}/I_{in}$$

Higher magnification means higher fraction of power is extracted.

Also, the higher is a magnification, the near-field output beam will then be as nearly fully illuminated as possible, and the far-field beam pattern will have as much energy as possible concentrated in the main central lobe.

Fresnel numbers

Fresnel number describes a diffractional behavior of a beam, which passed aperture :

$$N_F = a^2/L\lambda$$

This term is also applied to optical cavities. Siegman introduced so called collimated Fresnel number for unstable cavities :

$$N_c = \frac{(Ma)^2}{MB\lambda_0} = \frac{Ma^2}{B\lambda_0}$$

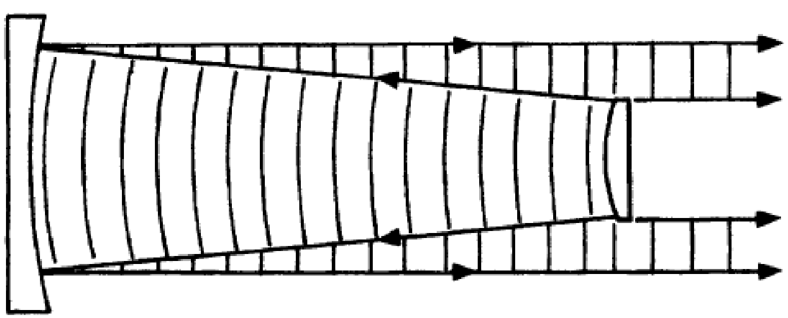
N_c determines amount of Fresnel diffraction ripples on the wavefront.

Siegman also introduced N_{eq}, which is proportional to N_c :

$$N_{eq} = \frac{M^2-1}{2M}\frac{a^2}{B\lambda_0} = \frac{M^2-1}{2M^2} \times N_c$$

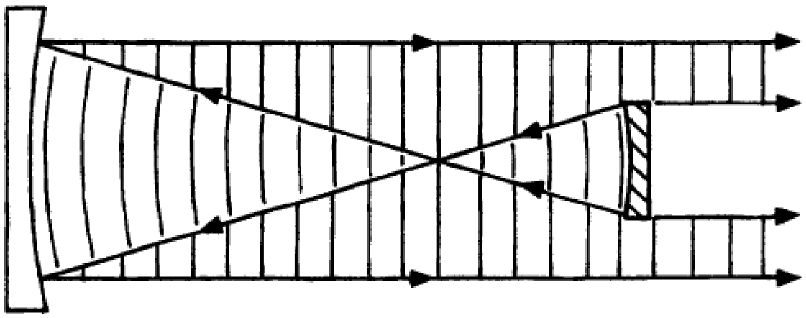
Geometrical description

Can be classified into positive and negative branch resonators



Positive branch resonator

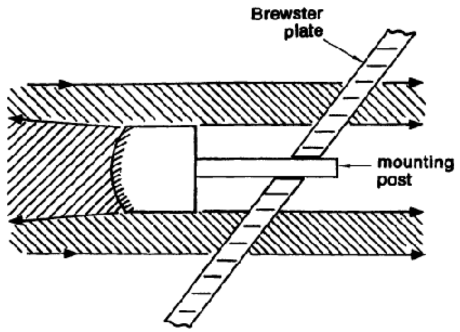
$$M = \frac{A+D}{2} > 1$$

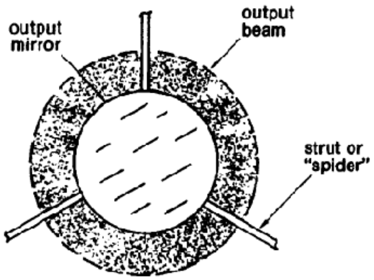


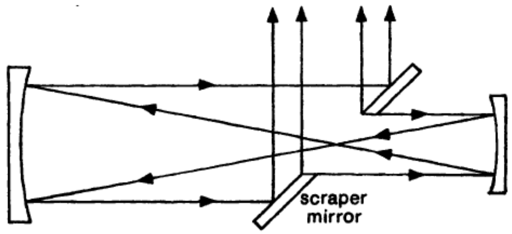
Negative branch resonator

$$M = \frac{A+D}{2} < -1$$

Output coupling methods





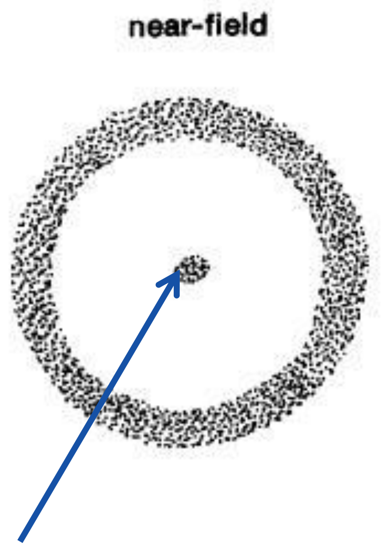


Brewster plate mount

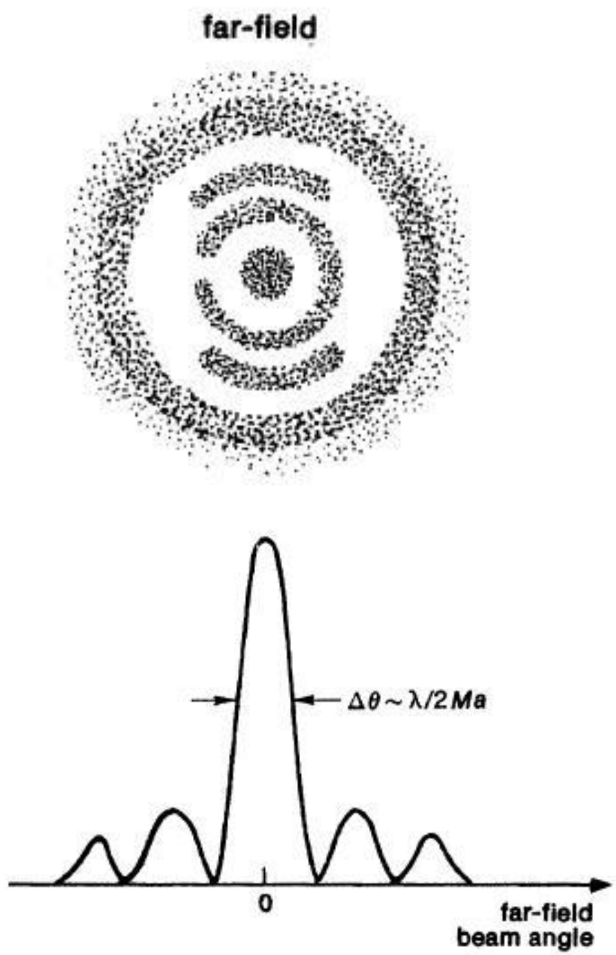
"Spider" mount

"Scraper mirror"

Output beam pattern



Very bright Arago spot due to diffraction on output mirror edge



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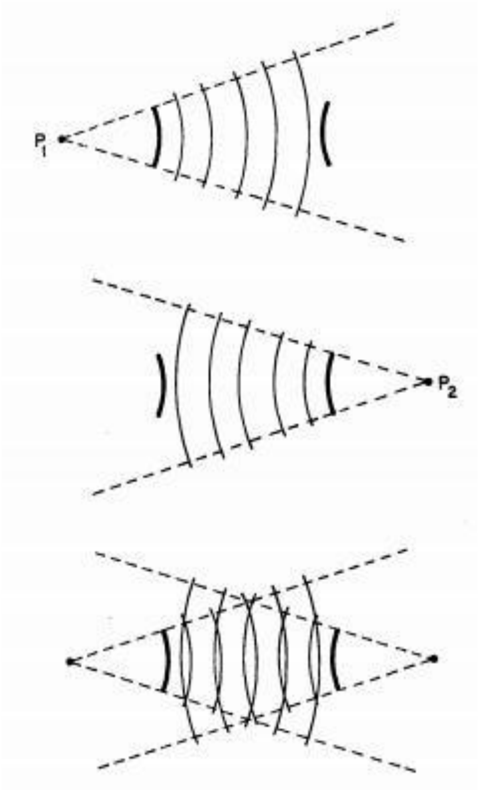
Mode analysis



Simple spherical wave analysis form a picture of lowest-order unstable resonator mode.

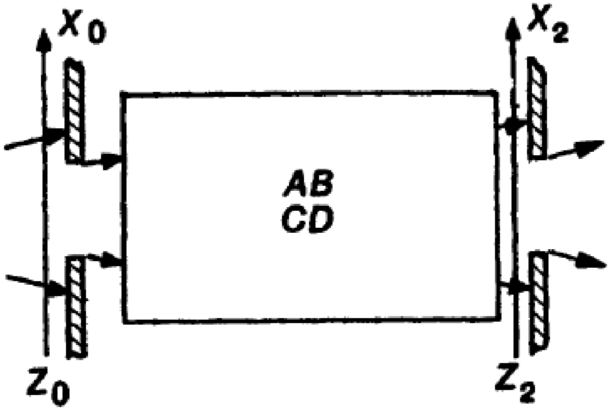
Distance between P1 and P2 must be chosen in a way that spherical wave after 1 roundtrip recreates initial wave.

Then the mode is self-consistent.

Mode analysis

Round-trip Huygens´ integral :

$$\tilde{u}_2(x_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0)\tilde{u}_0(x_0) \exp\left[-j\frac{\pi}{B\lambda_0}\left(Ax_0^2 - 2x_2x_0 + Dx_2^2\right)\right] dx_0$$



ABCD ray matrix model

Canonical formulation

Convertion of Huygens´ integral to canonical form begins with rewriting input (u_0) and output (u_2) waves into :

$$\tilde{u}_0(x_0) \equiv \tilde{v}_0(x_0) \times \exp\left[+j\frac{\pi(A-M)x_0^2}{B\lambda_0}\right]$$

$$\tilde{u}_2(x_2) \equiv \tilde{v}_2(x_2) \times \exp\left[-j\frac{\pi(D-1/M)x_2^2}{B\lambda_0}\right]$$

This is physically equivalent to extraction of the spherical curvature of unstable resonator modes, conversion of magnifying wavefronts to collimated wavefronts

Canonical formulation

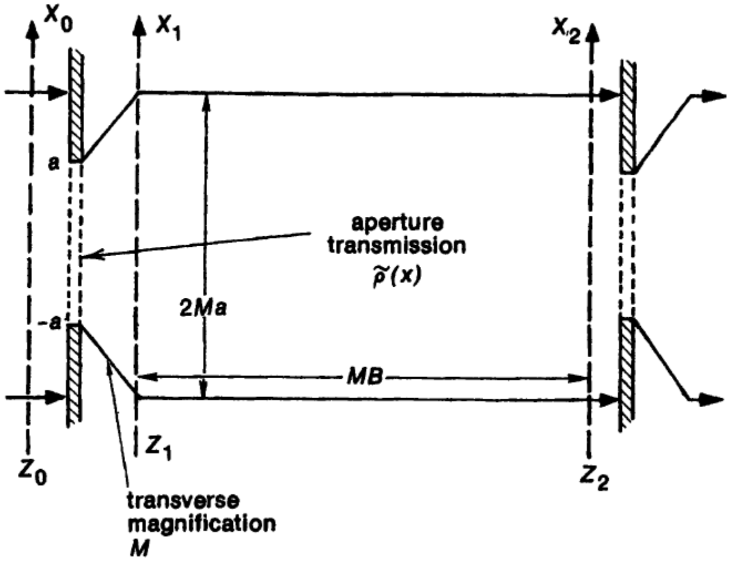
Huygens´ integral then turns into :

$$\tilde{v}_2(x_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0)\tilde{v}_0(x_0) \exp\left[-j\frac{\pi}{B\lambda_0}\left(Mx_0^2 - 2x_2x_0 + x_2^2/M\right)\right] dx_0$$

This integral corresponds to propagation through a simple collimated telescopic system with a ray matrix of a form :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \equiv \begin{bmatrix} M & B \\ 0 & 1/M \end{bmatrix} \equiv \begin{bmatrix} 1 & MB \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} M & 0 \\ 0 & 1/M \end{bmatrix}$$

Canonical formulation



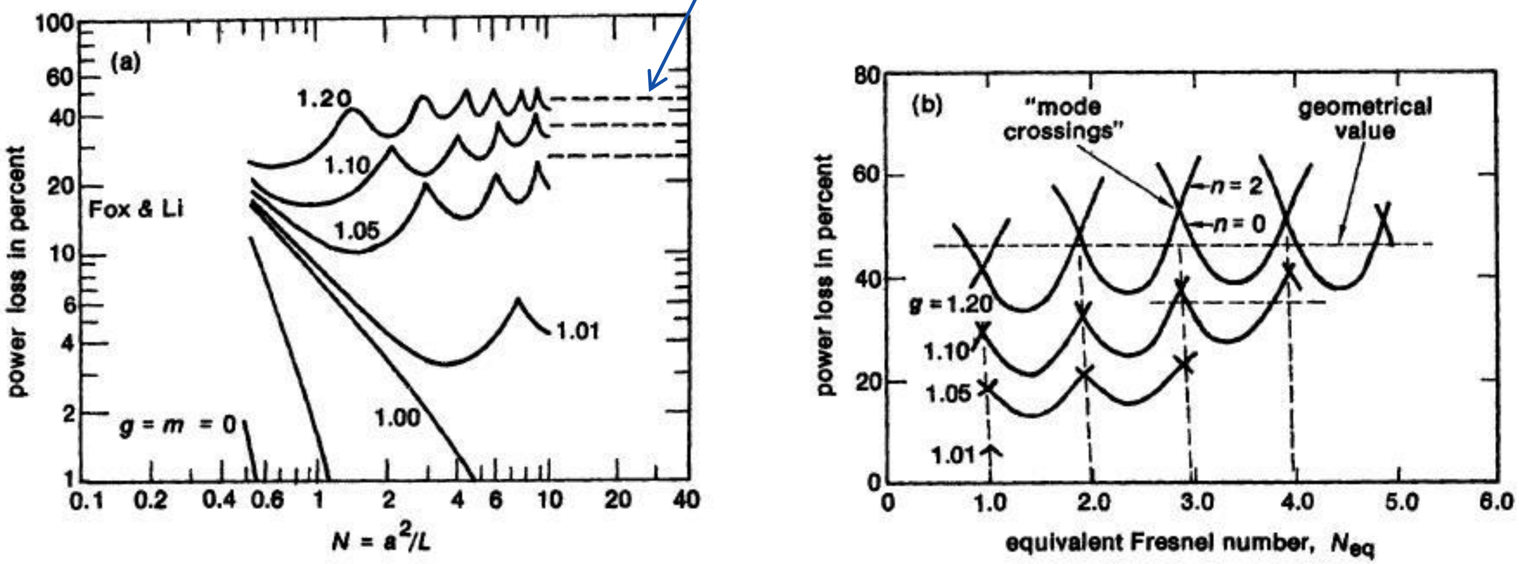
The system then can be expressed as the matrix product of zero-length telescope with magnification M and a free-space section of length MB

Reference plane is moved to magnified input plane z_1, so after change of variables $x_1=Mx_0$ we have free-space Huygens´ integral :

$$\tilde{v}_2(x_2) = \sqrt{\frac{j}{MB\lambda_0}} \int_{-Ma}^{Ma} \frac{\tilde{\rho}(x_1/M)\tilde{v}_0(x_1/M)}{M^{1/2}} \exp\left[-j\frac{\pi(x_1-x_2)^2}{MB\lambda_0}\right].dx_1$$

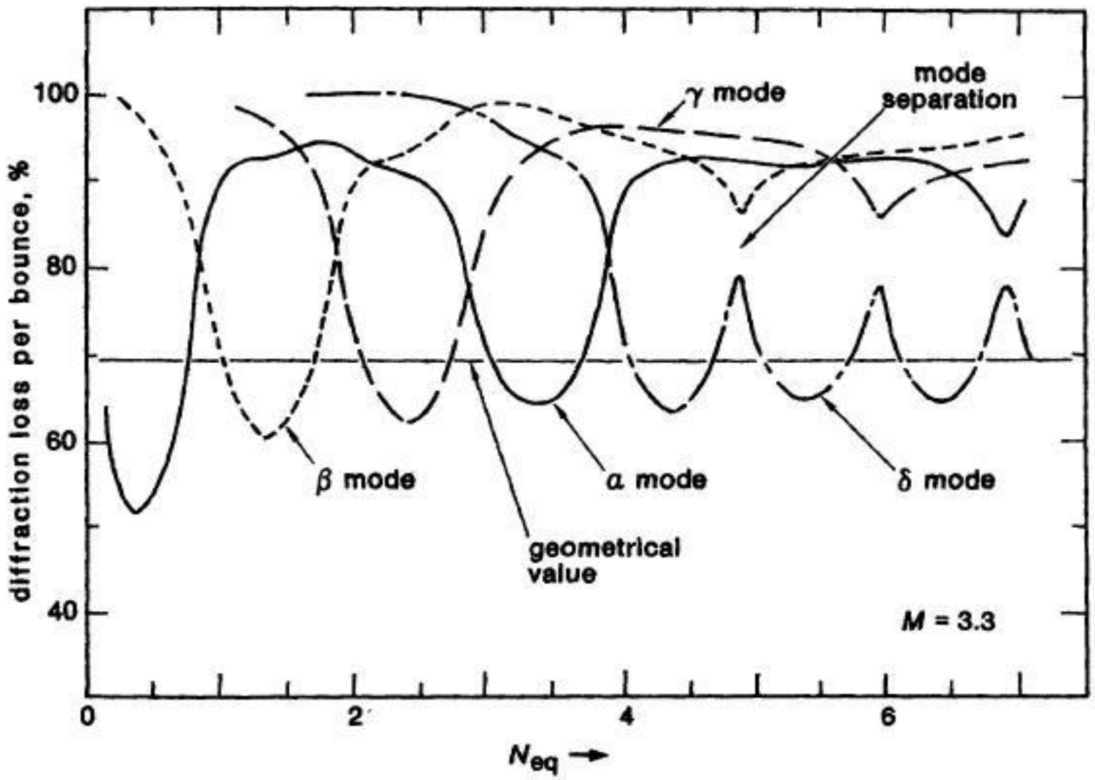
Loss calculations

Geometrical prediction : loss = 1 – 1/M



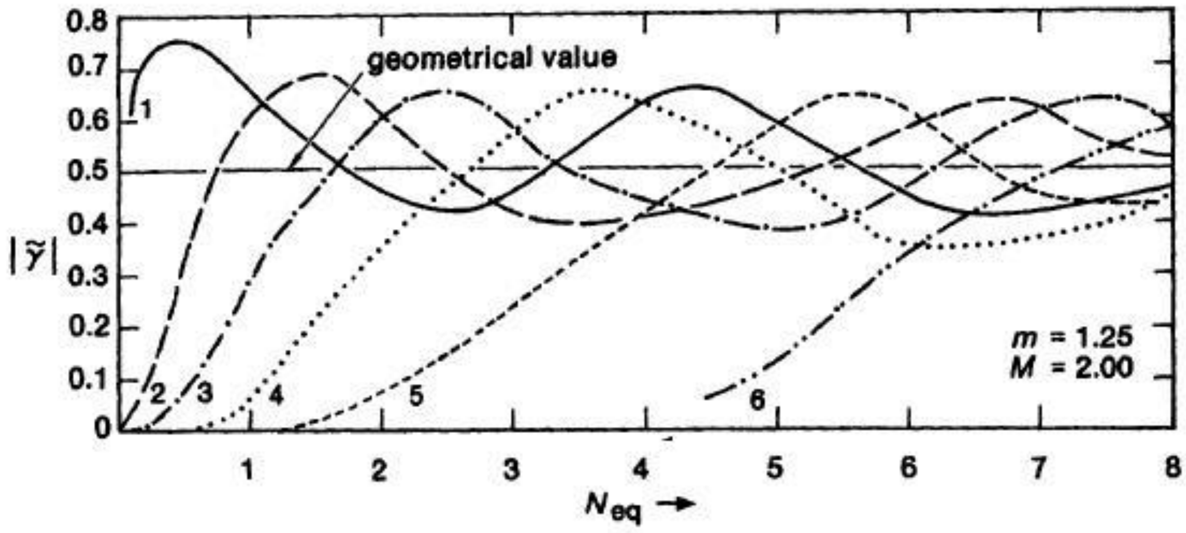
Low-loss behavior ”travels” from 1 mode to another

Loss calculations



Feature of rectangular unstable resonators!
Mode separation at high M and high N_{eq}

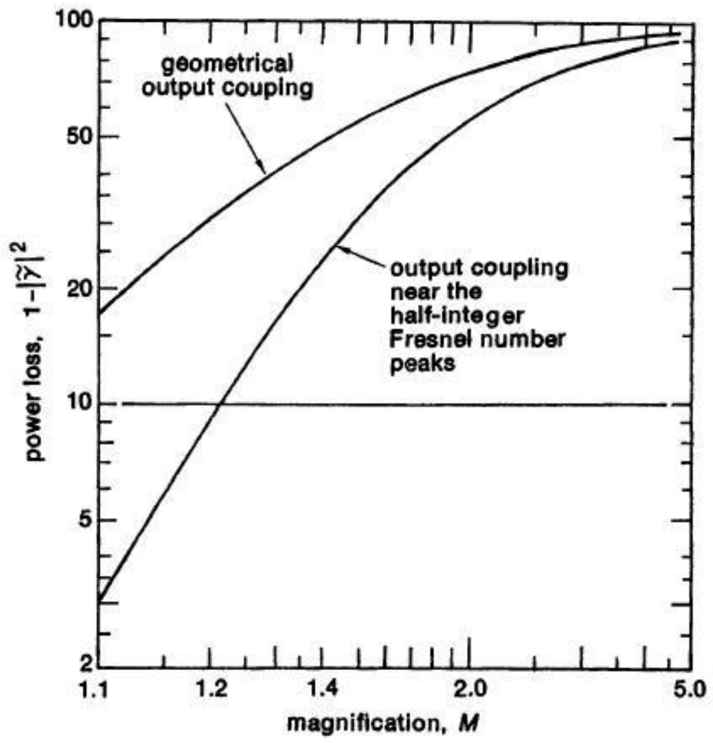
Eigenvalues for circular-mirror resonators



Majority of modes appear at low N_{eq} values , additional modes appear from high loss region with N_{eq} increasing.

Points on halfway (peaks) between mode crossing look like optimal N_{eq} values for unstable resonator operation.

Output coupling approximation



Geometrically predicted diffraction losses are always higher than ones calculated for optimum operation points.

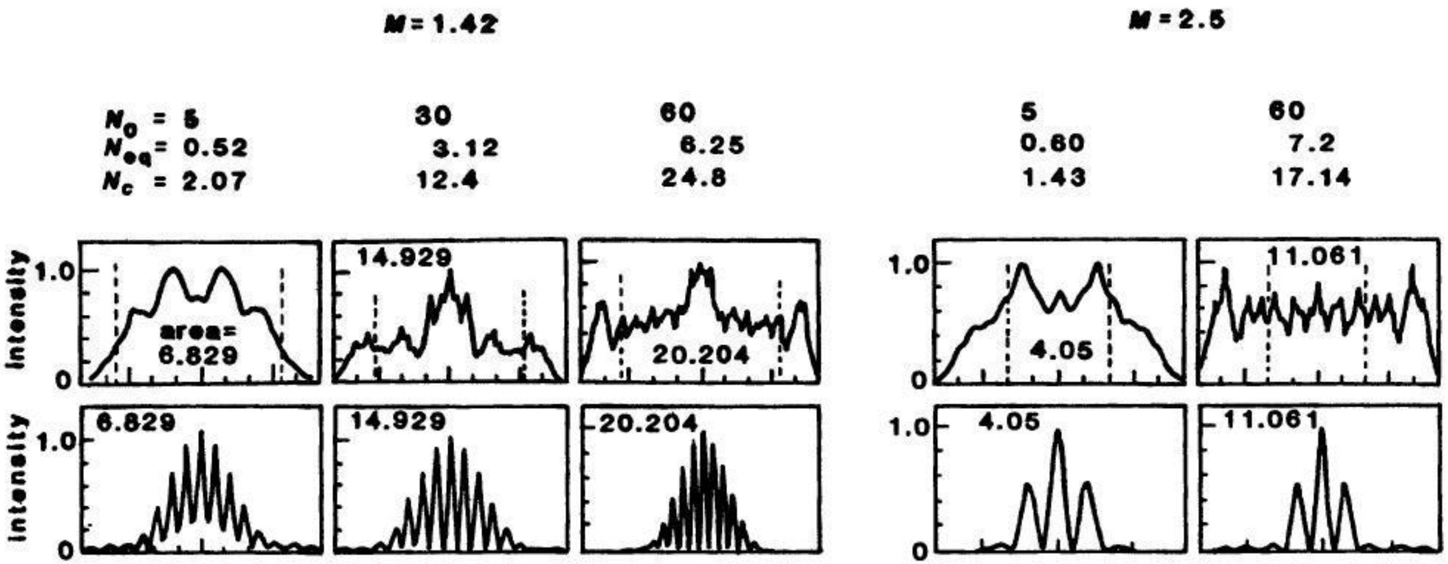
Geometric eigenvalue magnitude :

$$\tilde{\gamma}_{\mathbf{geom}} = 1/M$$

Eigenvalue at optimum peaks :

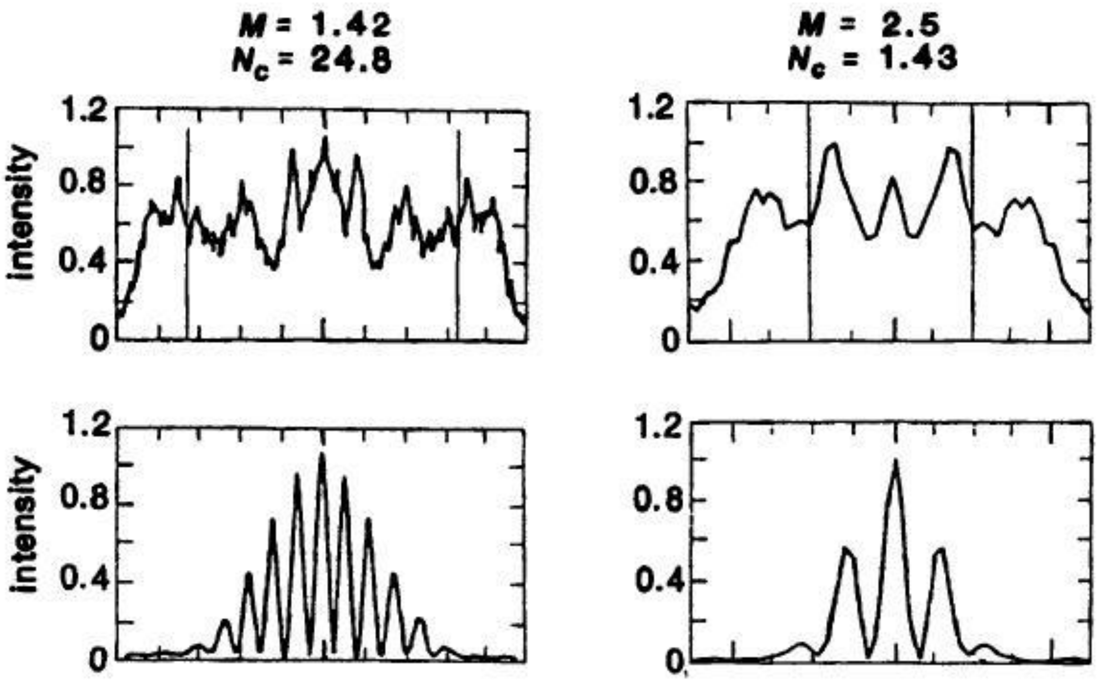
$$\tilde{\gamma}_{\mathbf{peak}} \approx \sqrt{\frac{2M^2-1}{M^4}}$$

Mode patterns



- mode shape changes from nearly Gaussian (N_{eq}<=1) to square-like (N_{eq}>>1)
- increasing N_c induces more Fresnel ripples
- higher M gives more power concentrated in central peak of far-field beam pattern

Loaded mode patterns



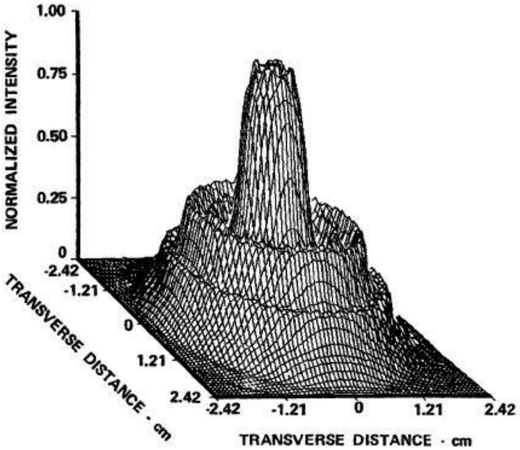
Some peaks in the near-field decreased in amplitude due to local gain saturation

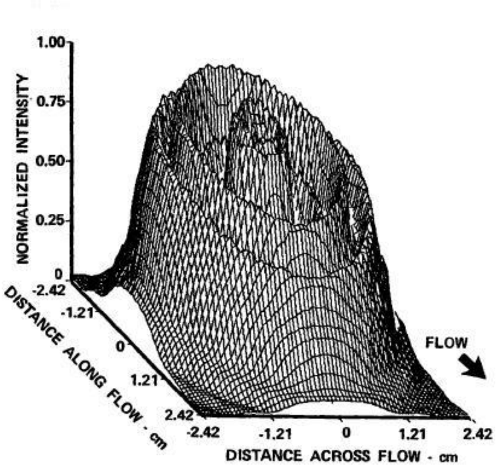
Numerical simulations

Parameters included :

• beam propagation, modified by mirror distortion, diffraction on edges, internal phase perturbations, etc

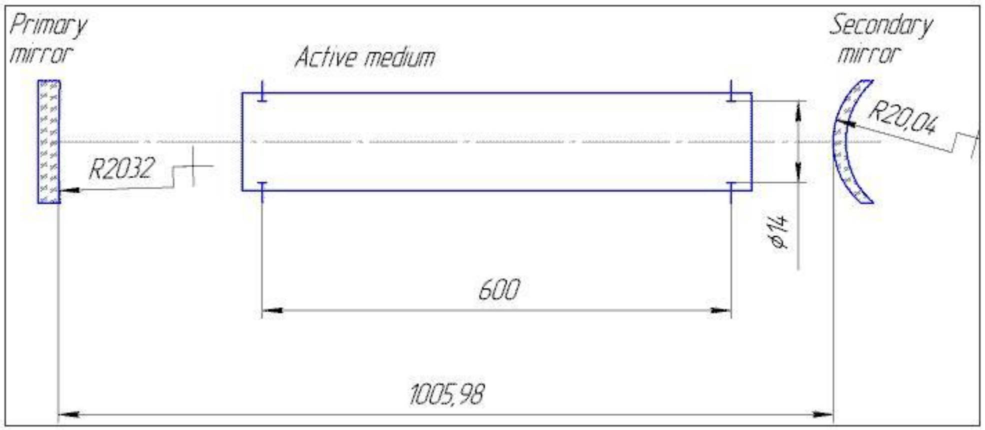
• gain medium characteristics, influenced by heating, saturation, repumping, possible chemical reactions and other effects

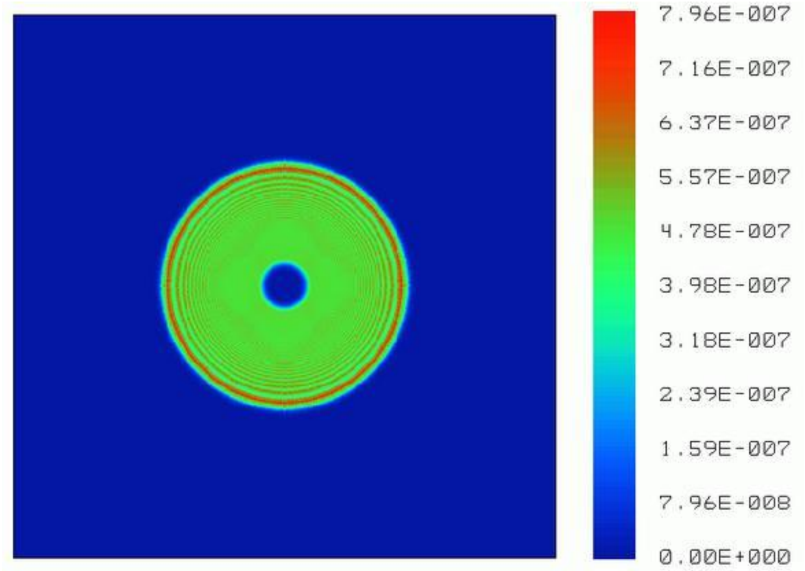




Near-field : unloaded and loaded simulation

Numerical simulations





(c) Zemax

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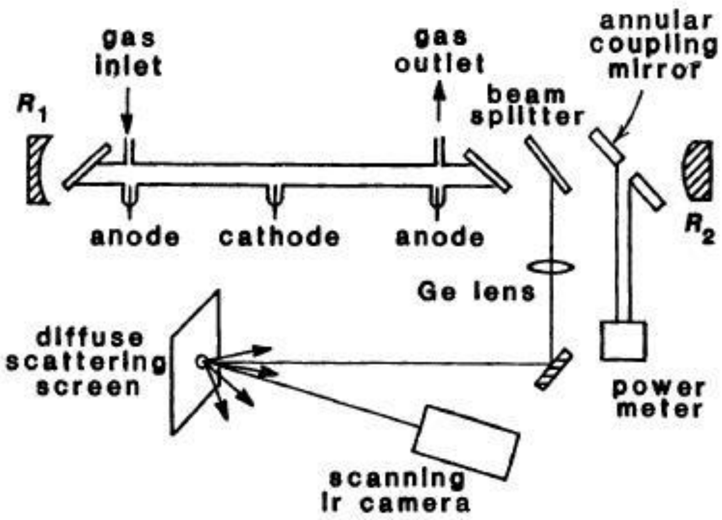
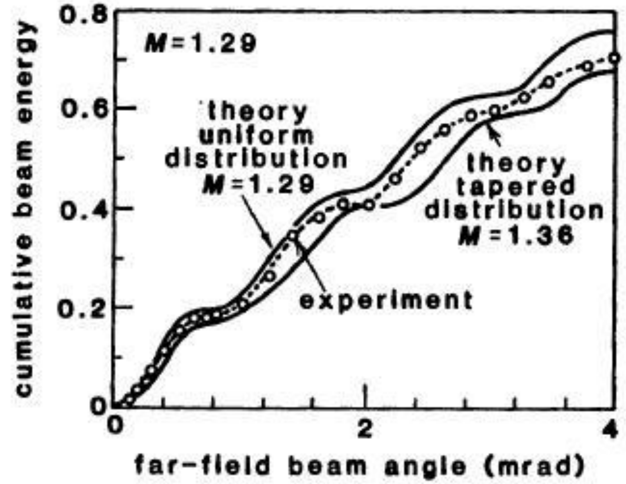
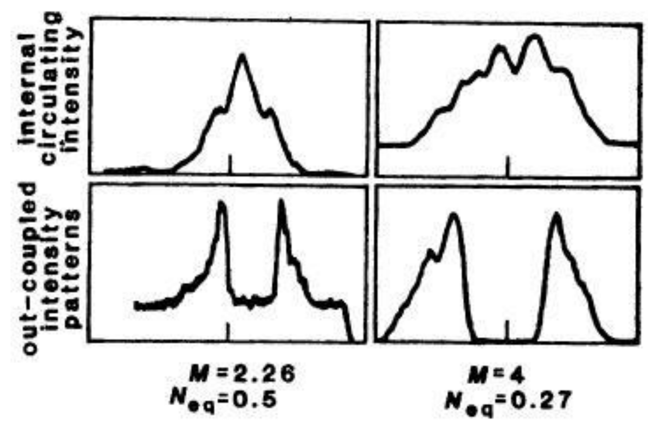
Experimental results







10.6 µm CO_2 laser with 2 different scraper coupling mirrors

Small deviation from theory for both near-field and far-field beam profile

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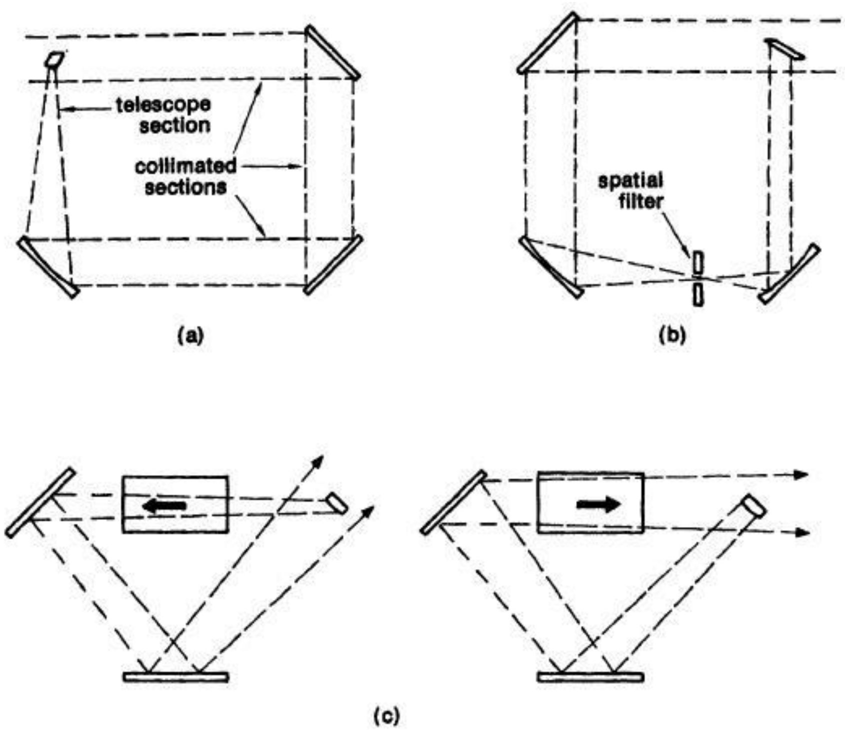
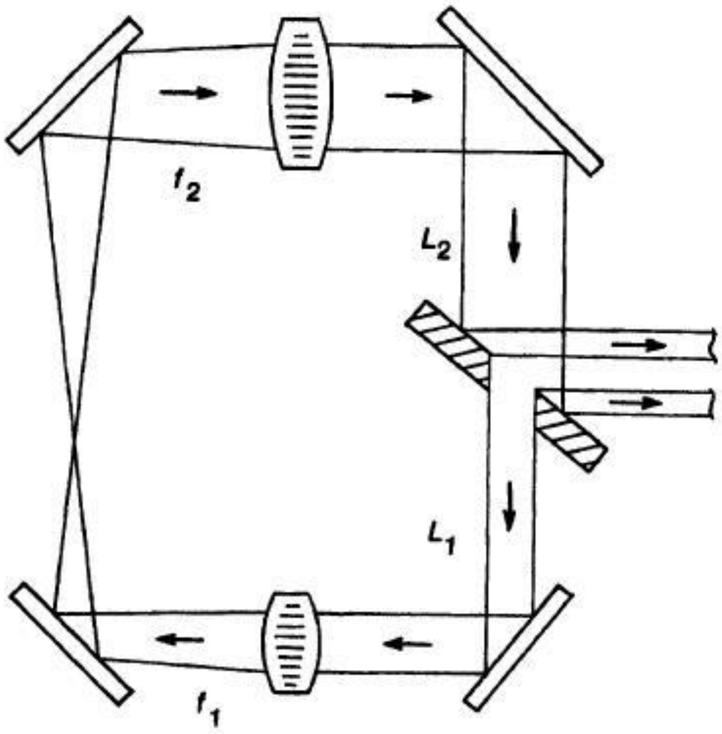
Ring unstable resonators



Possibility of unidirectional beam propagation : no spatial hole burning!

Self-imaging unstable resonators



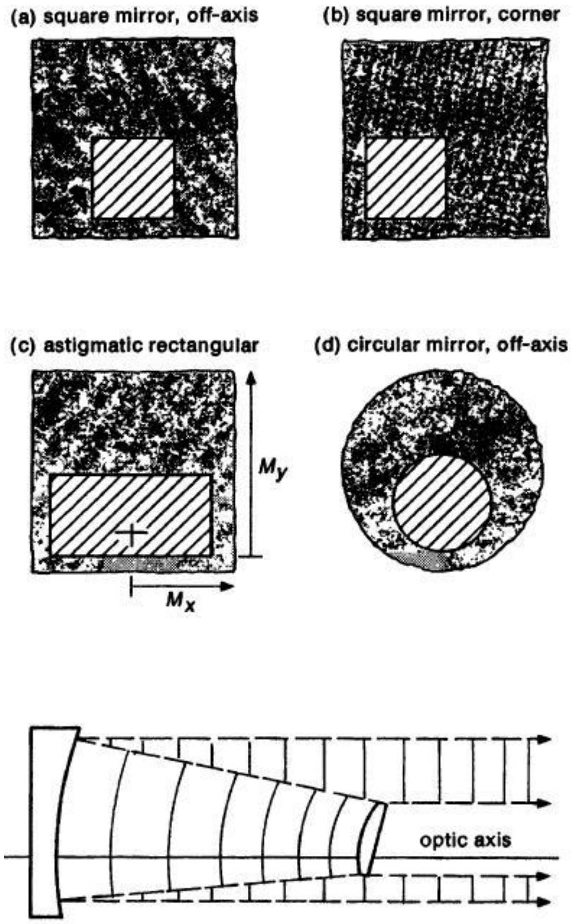
The system images magnified version of coupling aperture onto itself after each round-trip.

If self-imaging condition

$$f_1 + f_2 = ML_1 + L_2/M$$

is satisfied, then B=0 in cavity descriptive matrix, then each round-trip has zero effective propagation length, so the resonator has infinitely high Fresnel numbers, $\mathbf{N_c = N_{eq} = \infty}$

Off-axis unstable resonators

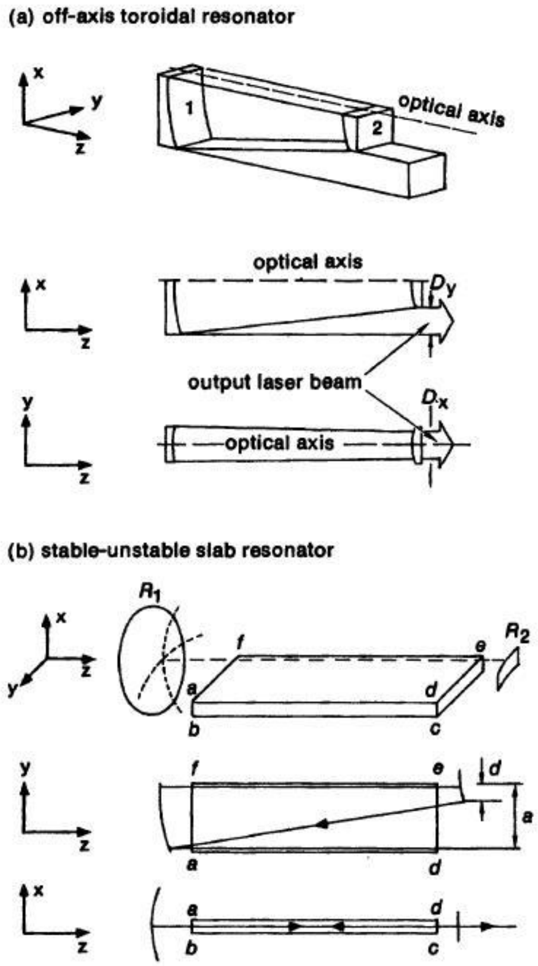


Designed for more uniform near-field pattern and to increase intensity in the central lobe of far-field pattern.

With mirror misaligned, system can be assumed to have 2 half-resonators with same M but different N numbers, this modifies near-field pattern.

When rectangular astigmatism introduced, there are 2 half-resonators with different M but same N numbers, then far-field pattern is modified.

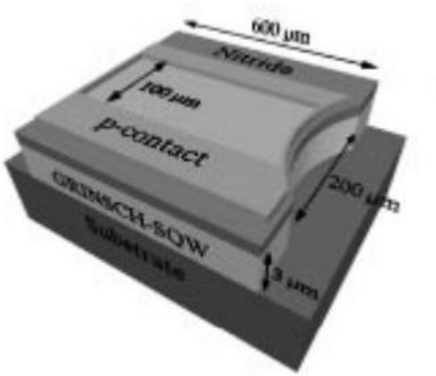
Stable-unstable resonators

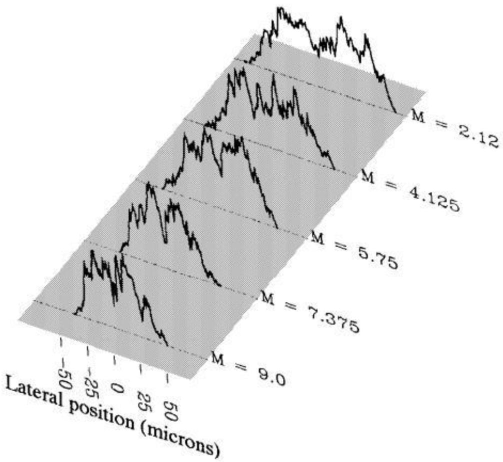


In such systems gain medium geometry exhibits high N numbers in 1 transverse direction, and low N numbers in another transverse direction (thin slab or sheet).

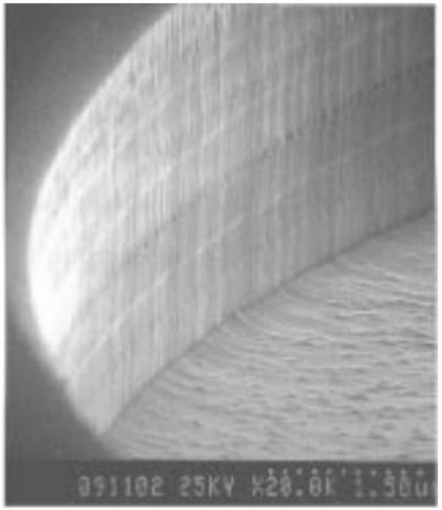
The cacity is then stable in 1 direction and unstable in another.

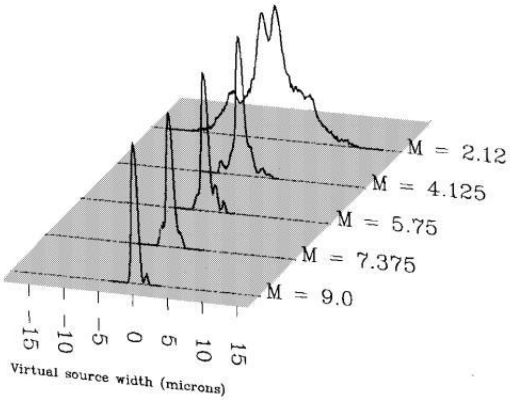
Unstable resonators in semiconductor lasers





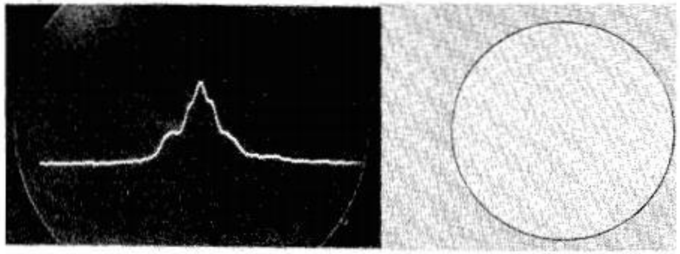
(a)

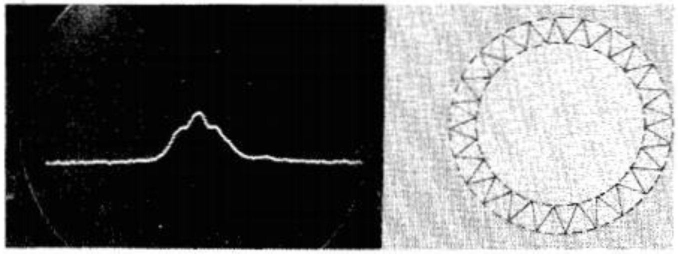


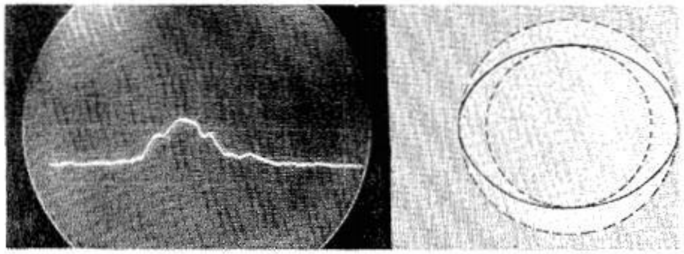


(c) Biellak et al.

Minimizing edge wave effects : aperture shaping

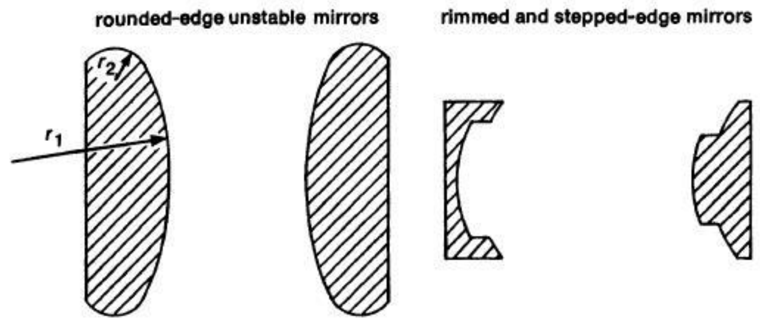

(a)

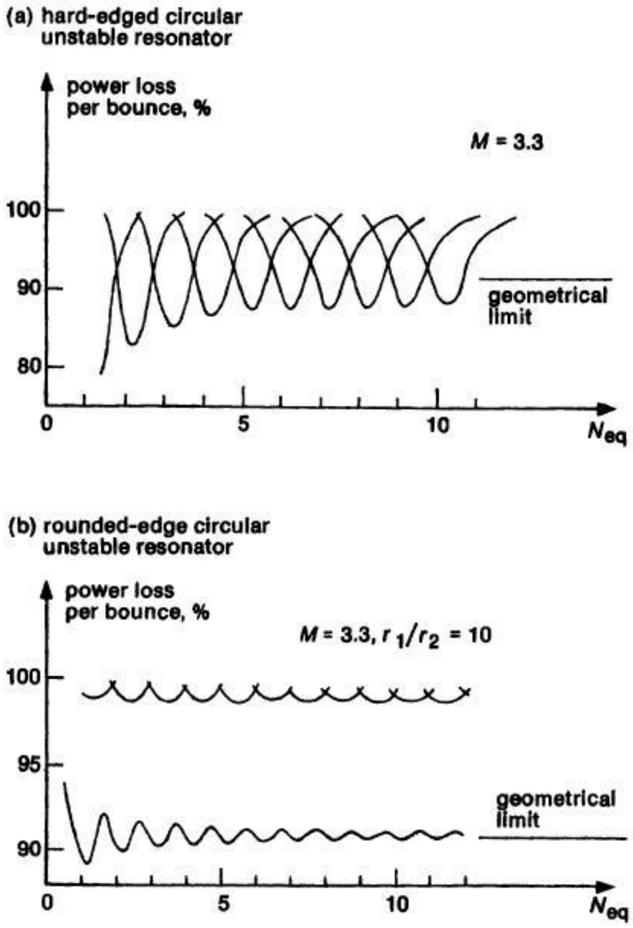

(b)


(c)

(c) Maunders et al.

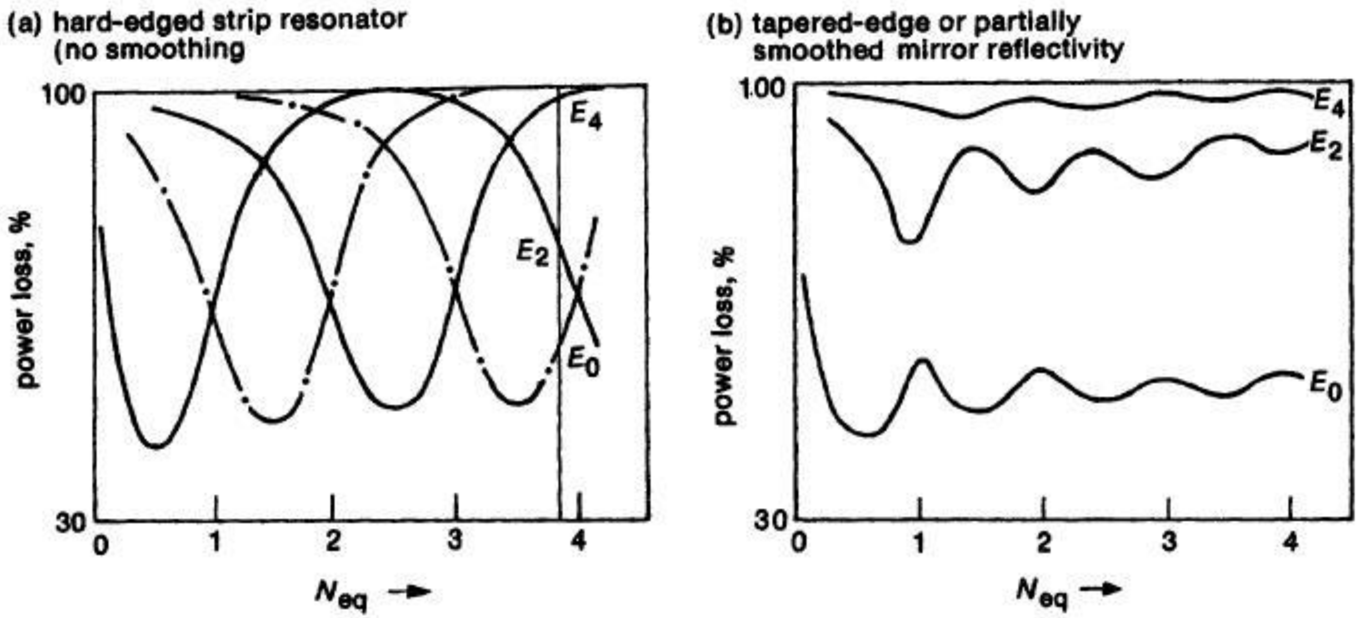
Minimizing edge wave effects : mirror tapering





Causes fundamental mode to separate from higher order modes due to reduced diffraction on mirror edges.

Variable reflectivity unstable resonators



Unstable cavity is combined with partially reflecting mirrors (Gaussian mirror, for example).
Control of mode behavior + better beam profile.

(c) Ananjev et al.

Conclusions

- Large controllable mode volume
- Controllable diffractive output coupling
- Good transverse mode discrimination
- All-reflective optics
- Automatically collimated output beams
- Easy to align
- Efficient power extraction
- Good far field beam patterns