Unstable optical resonators

Laser Physics course SK3410
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Outline

• Types of resonators
• Geometrical description
• Mode analysis
• Experimental results
• Other designs of unstable resonators
• Conclusions
Types of resonators

Stable

Beam is maintained inside limited volume of active medium

Unstable

Beam expands more and more with each bounce
Resonator stability

Stability condition:

\[ g_1 = 1 - \frac{L}{R_1}, \quad g_2 = 1 - \frac{L}{R_2} \]

\[ 0 \leq \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leq 1. \]
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Geometrical description

Magnification per roundtrip:

\[ M = \frac{I_{\text{out}}}{I_{\text{in}}} \]

Higher magnification means higher fraction of power is extracted.

Also, the higher is a magnification, the near-field output beam will then be as nearly fully illuminated as possible, and the far-field beam pattern will have as much energy as possible concentrated in the main central lobe.
Fresnel numbers

Fresnel number describes a diffractional behavior of a beam, which passed aperture:

\[ N_F = \frac{a^2}{L\lambda} \]

This term is also applied to optical cavities. Siegman introduced so called collimated Fresnel number for unstable cavities:

\[ N_c = \frac{(Ma)^2}{MB\lambda_0} = \frac{Ma^2}{B\lambda_0} \]

\( N_c \) determines amount of Fresnel diffraction ripples on the wavefront.

Siegman also introduced \( N_{eq} \), which is proportional to \( N_c \):

\[ N_{eq} = \frac{M^2 - 1}{2M} \frac{a^2}{B\lambda_0} = \frac{M^2 - 1}{2M^2} \times N_c \]
Geometrical description

Can be classified into positive and negative branch resonators

Positive branch resonator

$$M = \frac{A+D}{2} > 1$$

Negative branch resonator

$$M = \frac{A+D}{2} < -1$$
Output coupling methods

Brewster plate mount  “Spider” mount  “Scraper mirror”
Output beam pattern

Very bright Arago spot due to diffraction on output mirror edge
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Mode analysis

Simple spherical wave analysis form a picture of lowest-order unstable resonator mode.

Distance between P1 and P2 must be chosen in a way that spherical wave after 1 roundtrip recreates initial wave.

Then the mode is self-consistent.
Mode analysis

Round-trip Huygens’ integral:

\[ \tilde{u}_2(x_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0)\tilde{u}_0(x_0) \exp \left[ -j \frac{\pi}{B\lambda_0} (Ax_0^2 - 2x_2x_0 + Dx_2^2) \right] dx_0. \]

ABCD ray matrix model
Canonical formulation

Conversion of Huygens’ integral to canonical form begins with rewriting input ($u_0$) and output ($u_2$) waves into:

$$\tilde{u}_0(x_0) \equiv \tilde{v}_0(x_0) \times \exp \left[ +j \frac{\pi(A - M)x_0^2}{B\lambda_0} \right]$$

$$\tilde{u}_2(x_2) \equiv \tilde{v}_2(x_2) \times \exp \left[ -j \frac{\pi(D - 1/M)x_2^2}{B\lambda_0} \right]$$

This is physically equivalent to extraction of the spherical curvature of unstable resonator modes, conversion of magnifying wavefronts to collimated wavefronts.
Huygens’ integral then turns into:

$$\tilde{v}_2(x_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0) \tilde{v}_0(x_0) \exp \left[ -j \frac{\pi}{B\lambda_0} \left( Mx_0^2 - 2x_2x_0 + x_2^2/M \right) \right] \, dx_0$$

This integral corresponds to propagation through a simple collimated telescopic system with a ray matrix of a form:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \equiv \begin{bmatrix} M & B \\ 0 & 1/M \end{bmatrix} \equiv \begin{bmatrix} 1 & MB \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} M & 0 \\ 0 & 1/M \end{bmatrix}$$
The system then can be expressed as the matrix product of zero-length telescope with magnification $M$ and a free-space section of length $MB$.

Reference plane is moved to magnified input plane $z_1$, so after change of variables $x_1 = Mx_0$ we have free-space Huygens' integral:

$$
\tilde{v}_2(x_2) = \sqrt{\frac{j}{MB\lambda_0}} \int_{-Ma}^{Ma} \frac{\rho(x_1/M)\tilde{v}_0(x_1/M)}{M^{1/2}} \exp \left[-j\frac{\pi(x_1 - x_2)^2}{MB\lambda_0}\right] \, dx_1
$$
Loss calculations

Geometrical prediction: \( \text{loss} = 1 - \frac{1}{M} \)

Low-loss behavior "travels" from 1 mode to another
Loss calculations

Feature of rectangular unstable resonators!
Mode separation at high $M$ and high $N_{eq}$
Eigenvalues for circular-mirror resonators

Majority of modes appear at low $N_{eq}$ values, additional modes appear from high loss region with $N_{eq}$ increasing. Points on halfway (peaks) between mode crossing look like optimal $N_{eq}$ values for unstable resonator operation.
Output coupling approximation

Geometrically predicted diffraction losses are always higher than ones calculated for optimum operation points.

Geometric eigenvalue magnitude:

$$\tilde{\gamma}_{\text{geom}} = \frac{1}{M}$$

Eigenvalue at optimum peaks:

$$\tilde{\gamma}_{\text{peak}} \approx \sqrt{\frac{2M^2 - 1}{M^4}}$$
Mode patterns

- mode shape changes from nearly Gaussian (\(N_{eq} \leq 1\)) to square-like (\(N_{eq} \gg 1\))
- increasing \(N_c\) induces more Fresnel ripples
- higher \(M\) gives more power concentrated in central peak of far-field beam pattern
Loaded mode patterns

Some peaks in the near-field decreased in amplitude due to local gain saturation
Numerical simulations

Parameters included:

• beam propagation, modified by mirror distortion, diffraction on edges, internal phase perturbations, etc

• gain medium characteristics, influenced by heating, saturation, repumping, possible chemical reactions and other effects

Near-field: unloaded and loaded simulation
Numerical simulations

(c) Zemax
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Experimental results

10.6 µm CO₂ laser with 2 different scraper coupling mirrors

Small deviation from theory for both near-field and far-field beam profile
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Ring unstable resonators

Possibility of unidirectional beam propagation: no spatial hole burning!
Self-imaging unstable resonators

The system images magnified version of coupling aperture onto itself after each round-trip.

If self-imaging condition

\[ f_1 + f_2 = ML_1 + L_2/M \]

is satisfied, then \( B=0 \) in cavity descriptive matrix, then each round-trip has zero effective propagation length, so the resonator has infinitely high Fresnel numbers, \( N_c = N_{eq} = \infty \).
Off-axis unstable resonators

Designed for more uniform near-field pattern and to increase intensity in the central lobe of far-field pattern.

With mirror misaligned, system can be assumed to have 2 half-resonators with same M but different N numbers, this modifies near-field pattern.

When rectangular astigmatism introduced, there are 2 half-resonators with different M but same N numbers, then far-field pattern is modified.
Stable-unstable resonators

In such systems gain medium geometry exhibits high N numbers in 1 transverse direction, and low N numbers in another transverse direction (thin slab or sheet).

The cavity is then stable in 1 direction and unstable in another.
Unstable resonators in semiconductor lasers

(c) Biellak et al.
Minimizing edge wave effects: aperture shaping

(c) Maunders et al.
Minimizing edge wave effects : mirror tapering

Causes fundamental mode to separate from higher order modes due to reduced diffraction on mirror edges.
Variable reflectivity unstable resonators

Unstable cavity is combined with partially reflecting mirrors (Gaussian mirror, for example). Control of mode behavior + better beam profile.

(c) Ananjev et al.
Conclusions

- Large controllable mode volume
- Controllable diffractive output coupling
- Good transverse mode discrimination
- All-reflective optics
- Automatically collimated output beams
- Easy to align
- Efficient power extraction
- Good far field beam patterns