Pumping and population inversion
-
Laser amplification

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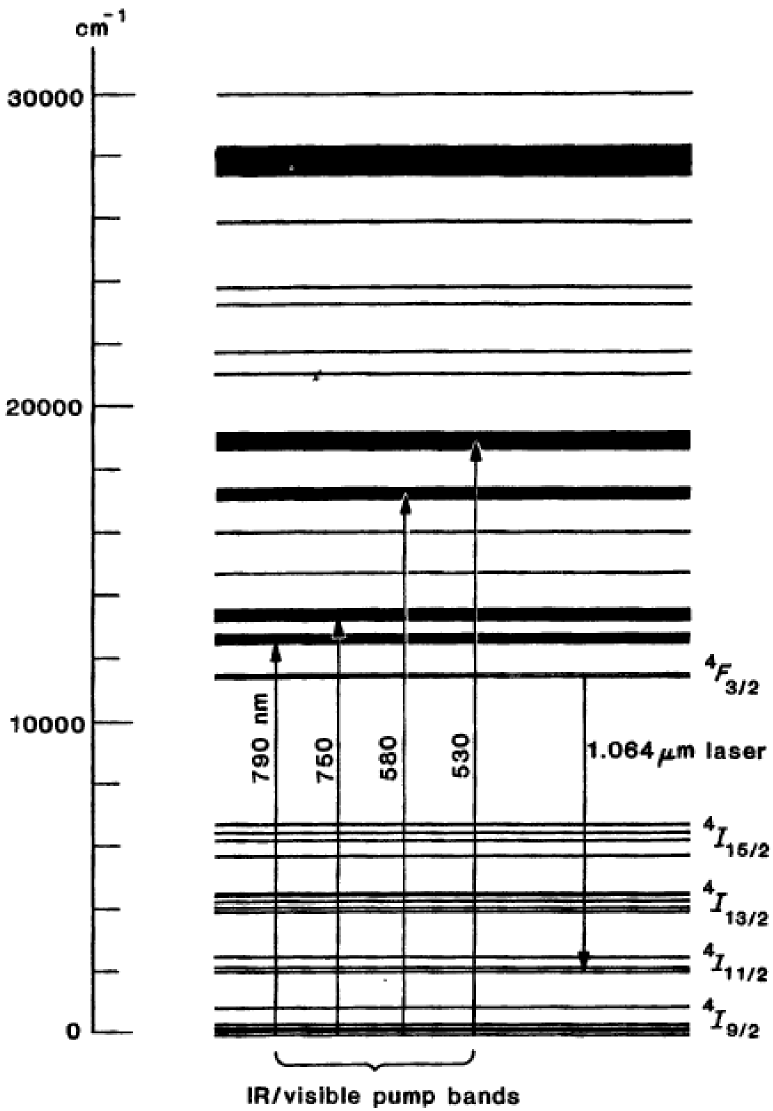
Contents

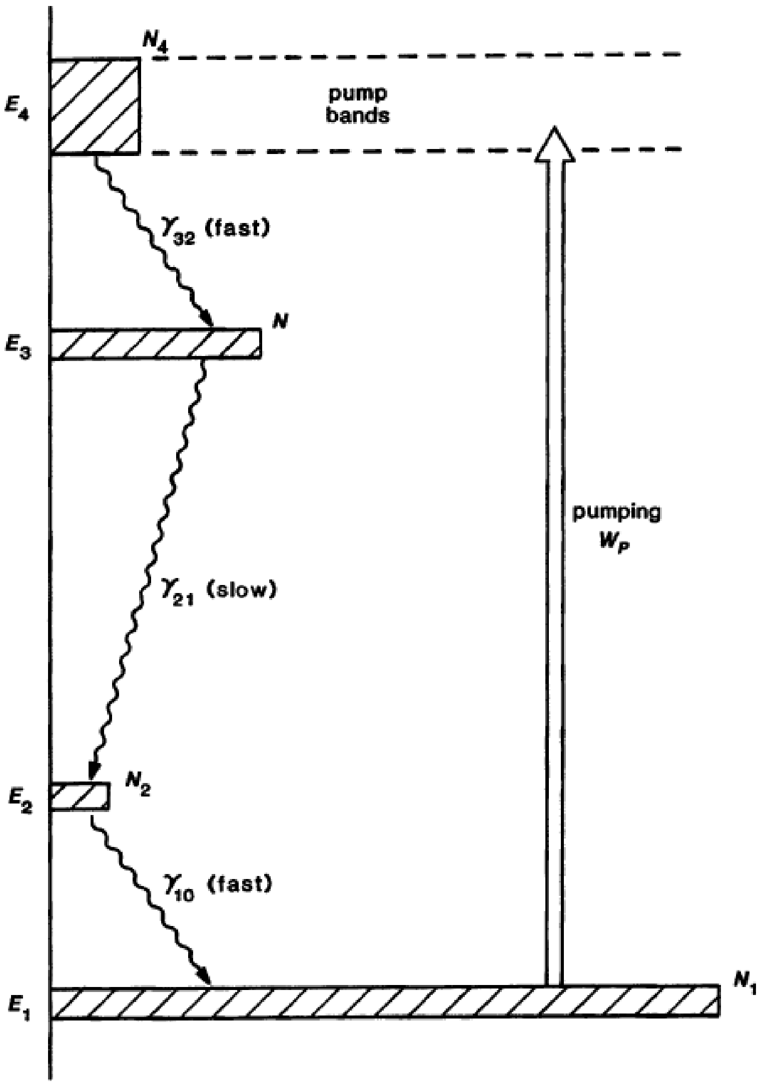
Atomic transitions



Energy-level diagram of Nd:YAG

Simplify into ->

4-level laser



Rate equations:

Pumping - Decay

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - N_4/\tau_{41}$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3}$$ Decay In/Out

$$\frac{dN_2}{dt} = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$ Same

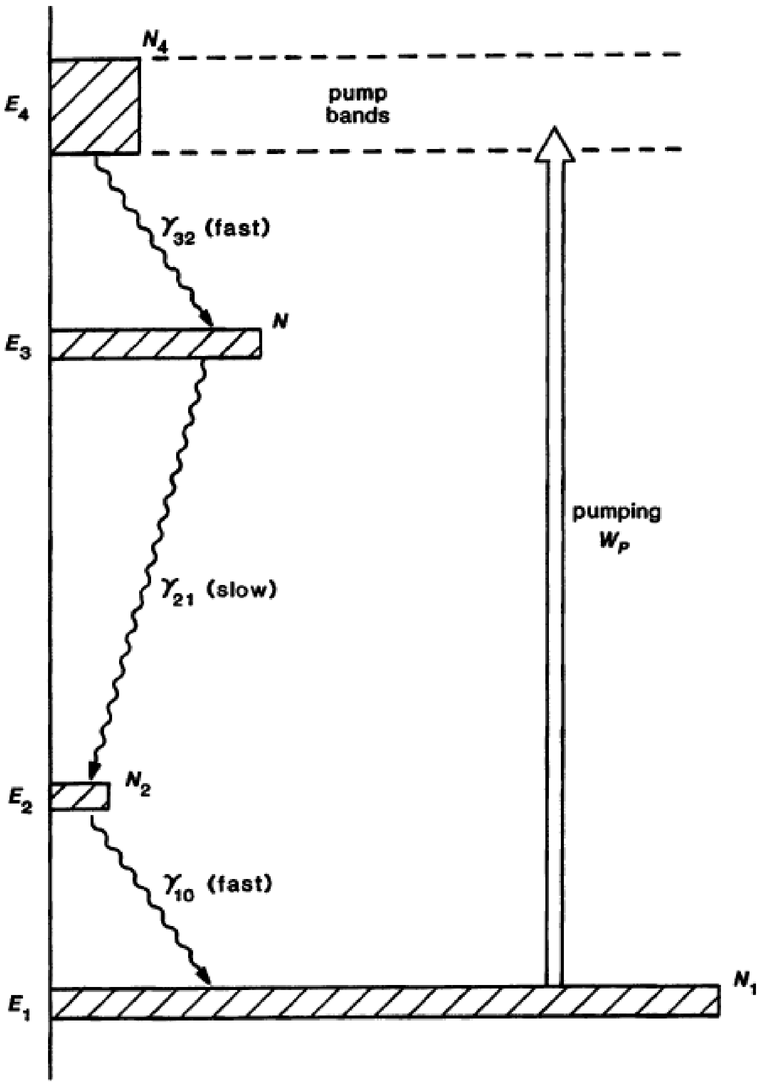
Atom conservation:

$N_1 + N_2 + N_3 + N_4 = N$

"Optical approximation",

$\hbar\omega/k_B T \ll 1$ No thermal occupancy

4-level laser



At steady state:

$$N_3 = \frac{\tau_3}{\tau_{43}} N_4$$

Define beta

$$N_2 = \left(\frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43}\tau_{21}}{\tau_{42}\tau_3}\right) N_3 \equiv \beta N_3$$

For a good laser: No direct decay into lev2

$\gamma_{42} \approx 0$ ($i.e.$ $\tau_{42} \to \infty$),

$$\to \quad \to \beta \approx \frac{\tau_{21}}{\tau_{32}}$$

Fluorescent quantum efficiency,

$$\eta \equiv \frac{\tau_4}{\tau_{43}} \cdot \frac{\tau_3}{\tau_{rad}}$$

Useful photons: from 4 -> upper laser

*

From upper laser that lase

4-level laser

Calculate the pop. Inv.

Population inversion,

$$\frac{N_3 - N_2}{N} = \frac{(1-\beta)\eta W_p \tau_{rad}}{1 + \left(1 + \beta + \frac{2\tau_{43}}{\tau_{rad}}\right)\eta W_p \tau_{rad}}$$

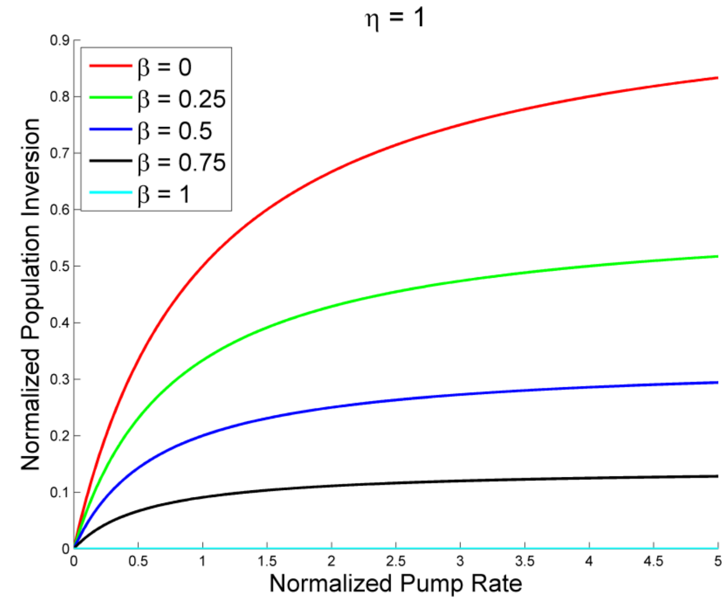
For a good laser:

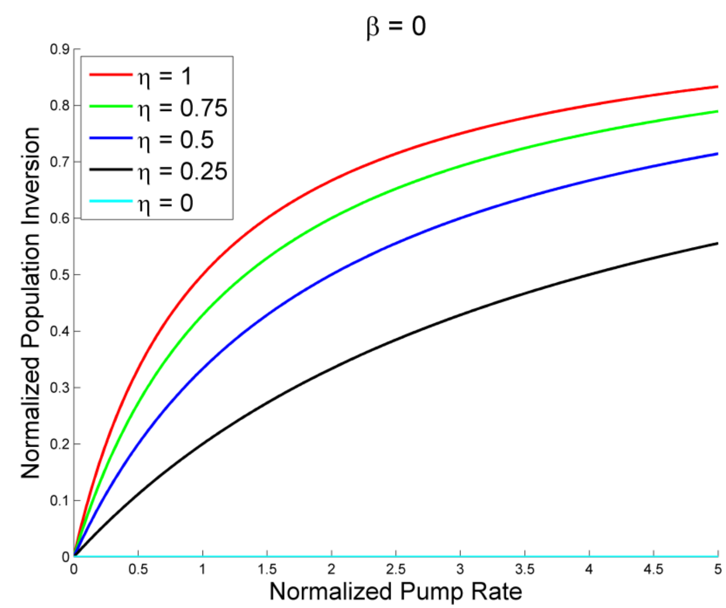
$\tau_{43} \ll \tau_{rad}$ - Short lev 4 lifetime

$\beta \approx \tau_{21}/\tau_{32} \rightarrow 0$ - Short lower lev lifetime

$\eta \rightarrow 1$ - High fluorescent quantum efficiency

$$\Rightarrow \frac{N_3 - N_2}{N} \approx \frac{W_p \tau_{rad}}{1 + W_p \tau_{rad}}$$ - Red curves

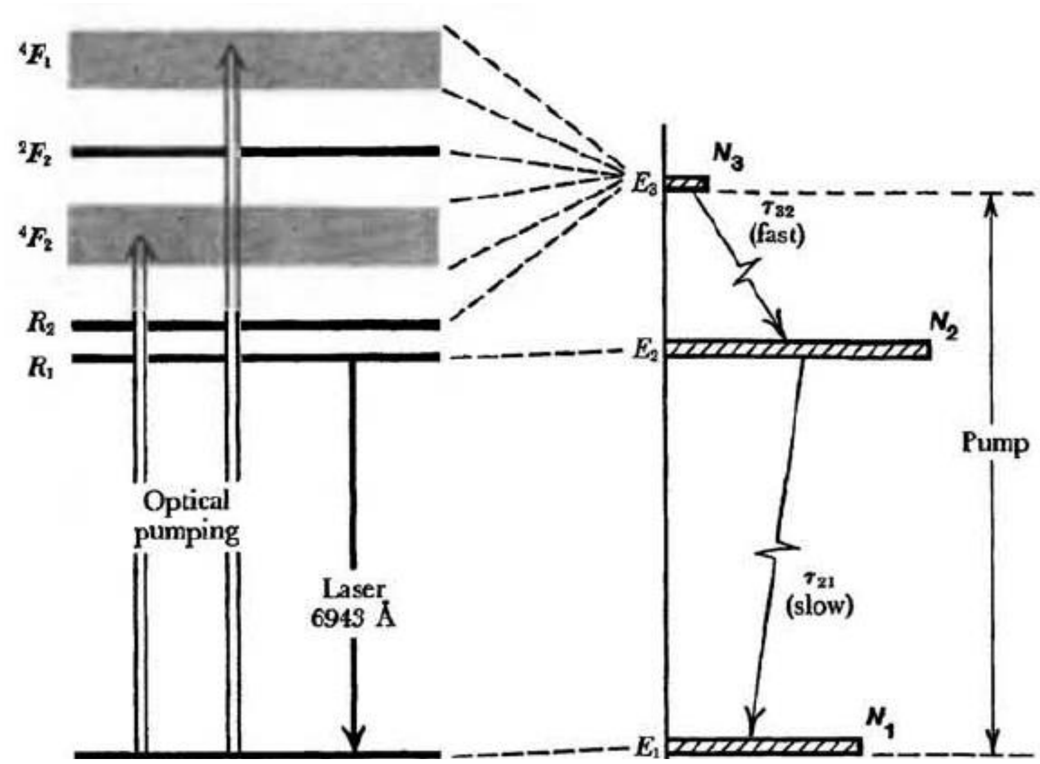




3-level laser

As before, for 3-level

BUT lower level is GROUND level



Rate equations: Pumping – decay

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - \frac{N_3}{\tau_3}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$ decays

Atom conservation:

$$N_1 + N_2 + N_3 = N$$ as before

$$\eta = \frac{\tau_3}{\tau_{32}}\frac{\tau_{21}}{\tau_{rad}}$$ As before

$$\beta = \frac{N_3}{N_2} = \frac{\tau_{32}}{\tau_{21}}$$ Different!

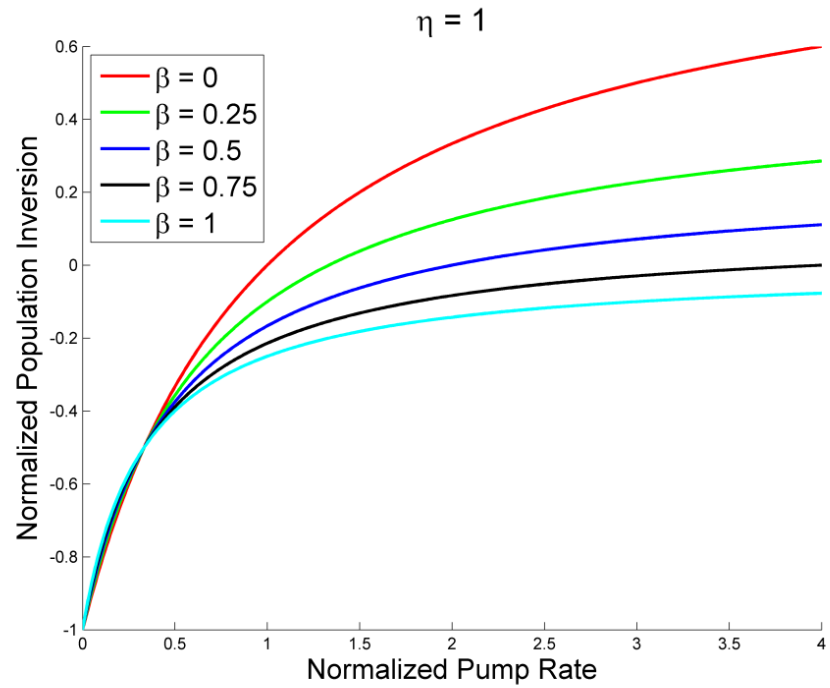
3-level laser

No pumping
NEGATIVE pop. Inv.

At steady state,

$$\frac{N_2 - N_1}{N} = \frac{(1-\beta)\eta W_p \tau_{rad} - 1}{(1+2\beta)\eta W_p \tau_{rad} + 1}$$

->



Requirements for pop. inversion:

$$\beta < 1$$ As before

$$W_p \tau_{rad} \geq \frac{1}{\eta(1-\beta)}$$ New

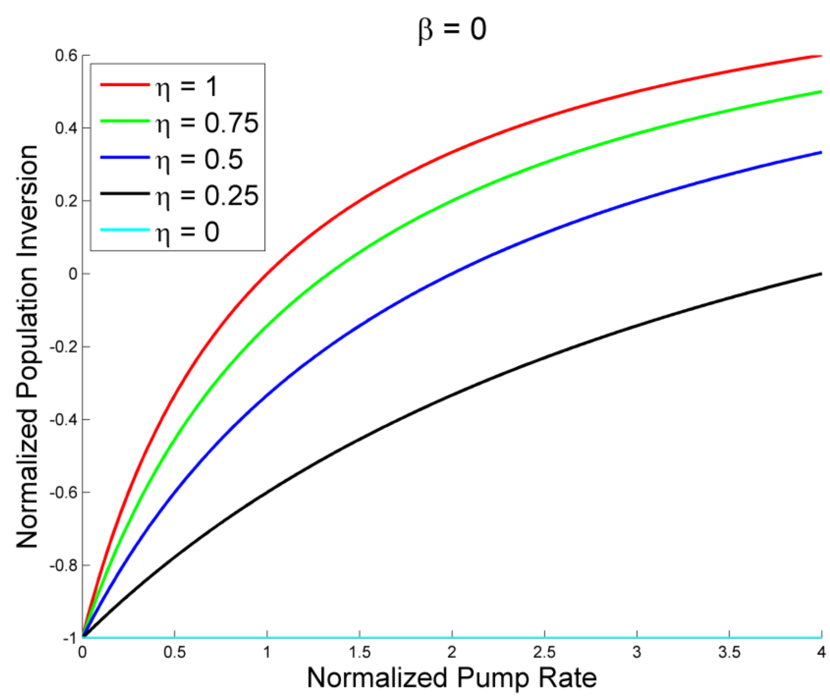
For a good laser,

$\beta \to 0$

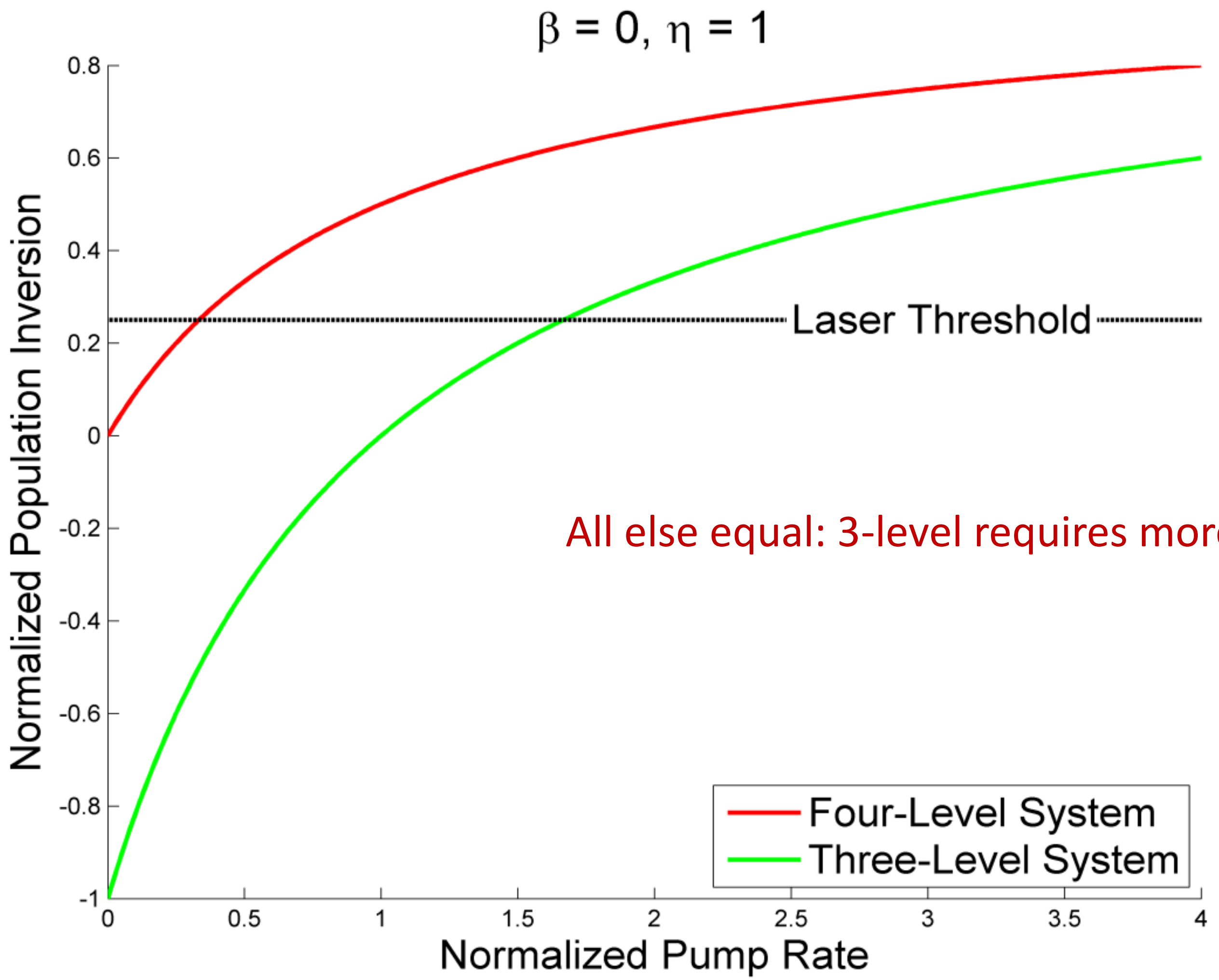
$\eta \to 1$

$$\frac{N_2 - N_1}{N} \approx \frac{W_p \tau_{rad} - 1}{W_p \tau_{rad} + 1}$$

Red curves

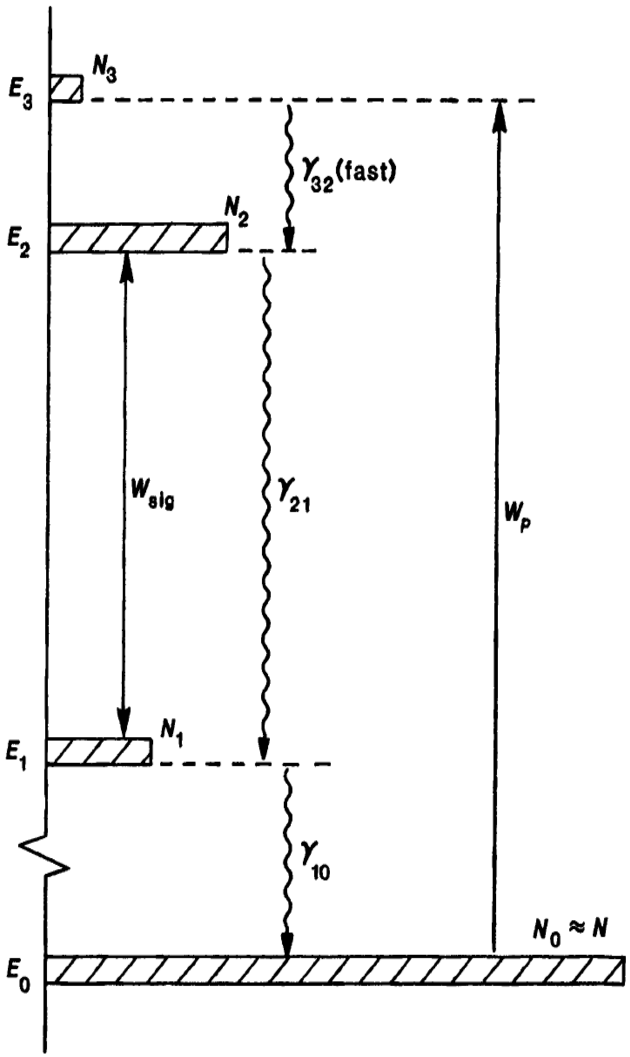


Population inversion



All else equal: 3-level requires more pumping

Upper-level laser



Lasing between two levels high above ground-level

$$\left.\frac{dN_3}{dt}\right|_{pump} = W_p(N_0 - N_3)$$

Pump into upper lev.

Assuming, $N_0 \approx N \gg N_3$ and pump efficiency, η_p,

$$\left.\frac{dN_3}{dt}\right|_{pump} \approx \eta_p W_p N \equiv R_p$$

most atoms in ground-state

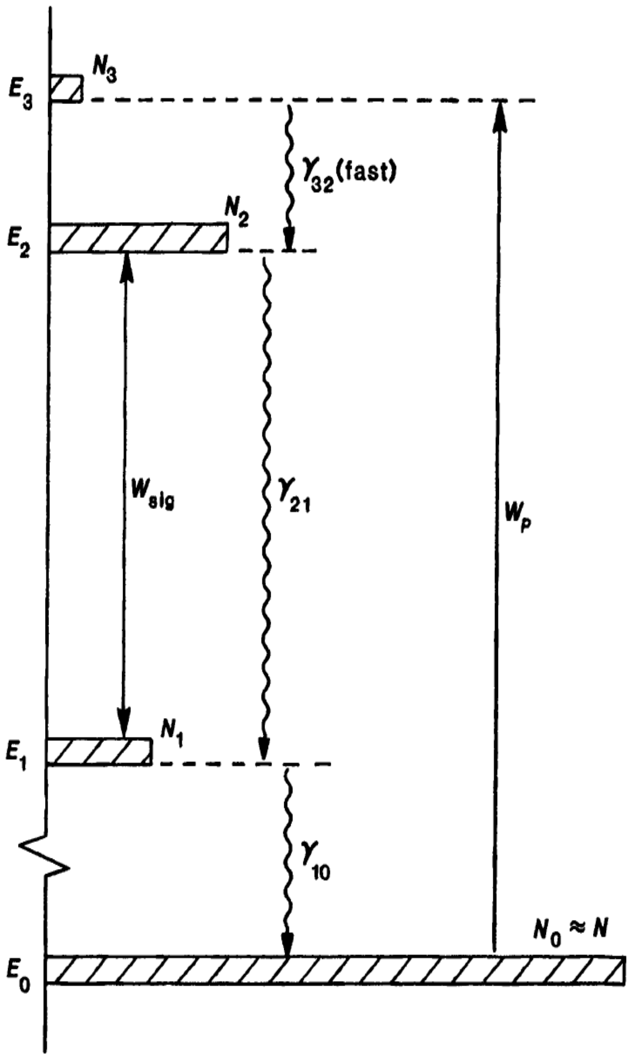
Rate equations:

Signal included

$$\frac{dN_2}{dt} = R_p - W_{sig}(N_2 - N_1) - \gamma_2 N_2$$

$$\frac{dN_1}{dt} = W_{sig}(N_2 - N_1) + \gamma_{21} N_2 - \gamma_1 N_1$$

Upper-level laser



At steady state:

$$N_1 = \frac{W_{sig} + \gamma_{21}}{W_{sig}(\gamma_1 + \gamma_{20}) + \gamma_1\gamma_2} R_p$$

$$N_2 = \frac{W_{sig} + \gamma_1}{W_{sig}(\gamma_1 + \gamma_{20}) + \gamma_1\gamma_2} R_p$$

No atom conservation!

For example, changing pump changes N

Upper-level laser

The pop. Inv. Saturates as the signal increases

Population inversion:

$$\Delta N_{21} = N_2 - N_1 = \left(\frac{\gamma_1 - \gamma_{21}}{\gamma_1 \gamma_2}\right) \cdot \frac{R_p}{1 + \left[\frac{\gamma_1 + \gamma_{20}}{\gamma_1 \gamma_2}\right] W_{sig}}$$

Define the small-signal population inversion, $\Delta N_0 = \frac{\gamma_1 - \gamma_{21}}{\gamma_1 \gamma_2} R_p$ and the effective recovery time, $\tau_{eff} = \tau_2 \left(1 + \frac{\tau_1}{\tau_{20}}\right)$ the expression becomes:

$$\Delta \mathrm{N}_{21} = \Delta N_0 \frac{1}{1 + W_{sig} \tau_{eff}}$$

For a good laser:

$\gamma_2 \approx \gamma_{21}$

$\gamma_{20} \approx 0$

$$\rightarrow \Delta \mathrm{N}_{21} \approx R_p (\tau_2 - \tau_1) \cdot \frac{1}{1 + W_{sig} \tau_2}$$

Prop. To pump-rate and lifetimes, saturation behavior

Upper-level laser

- Condition for obtaining inversion,

$$\tau_1/\tau_{21} < 1$$

i.e. **fast** relaxation from **lower level** and **slow** relaxation from **upper level**

- Small-signal gain,

$$\Delta N_0 \sim R_p \cdot \frac{\tau_2}{1-\tau_1/\tau_{21}}$$

i.e. **small-signal gain** is proportional to the **pump-rate** times a **reduced upper-level lifetime**

- Saturation behavior,

$$\Delta N_{21} = \Delta N_0 \cdot \frac{1}{1+W_{sig}\tau_{eff}}$$

i.e. the **saturation intensity** depends only on the **signal intensity** and the **effective lifetime**, **not** on the **pumping rate.**

Upper-level laser: Transient rate equation

As for instance before a Q-switched pulse

Assume: No signal ($W_{sig} = 0$), fast lower-level relaxation ($N_1 \approx 0$),

$$\frac{dN_2(t)}{dt} = R_p(t) - \gamma_2 N_2(t)$$

The upper level population becomes,

$$N_2(t) = \int_{-\infty}^{t} R_p(t') e^{-\gamma_2 (t-t')} dt'$$

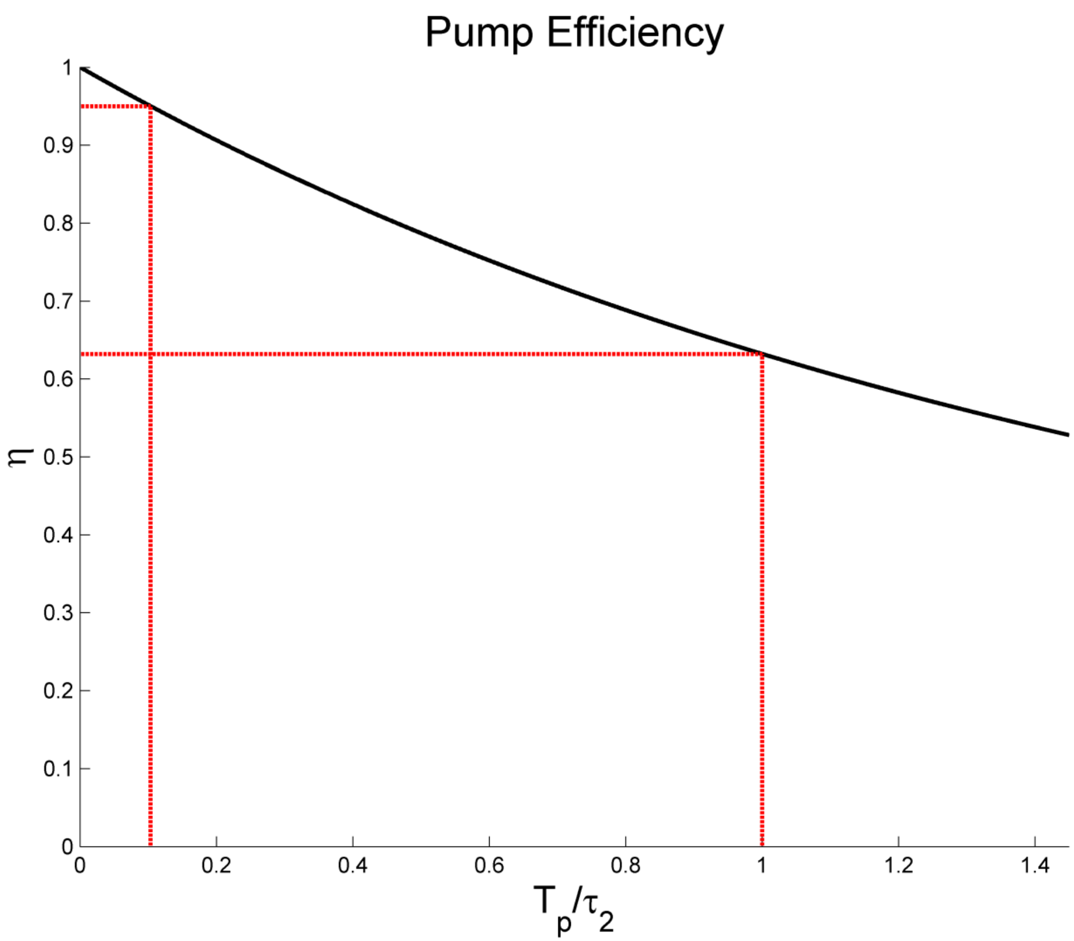
Applying a square pulse,

$$N_2(T_p) = R_{p0}\tau_2 (1 - e^{-T_p/\tau_2})$$

Define the pump efficiency,

$$\eta_p = \frac{N_2(t = T_p)}{R_{p0} T_p} = \frac{1 - e^{-T_p/\tau_2}}{T_p/\tau_2}$$

^Pop. In upper lev per pump-photon



3-level laser: pulses

3-level laser from prev,
no signal

Assume: No signal ($W_{sig} = 0$),
Fast upper-level relaxation ($\tau_3 \approx 0$),

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} \approx -W_p(t)N_1(t) + \frac{N_2(t)}{\tau}$$

$$\frac{d}{dt}\Delta N(t) = -\left[W_p(t) + \frac{1}{\tau}\right]\Delta N(t) + \left[W_p(t) - \frac{1}{\tau}\right]N$$

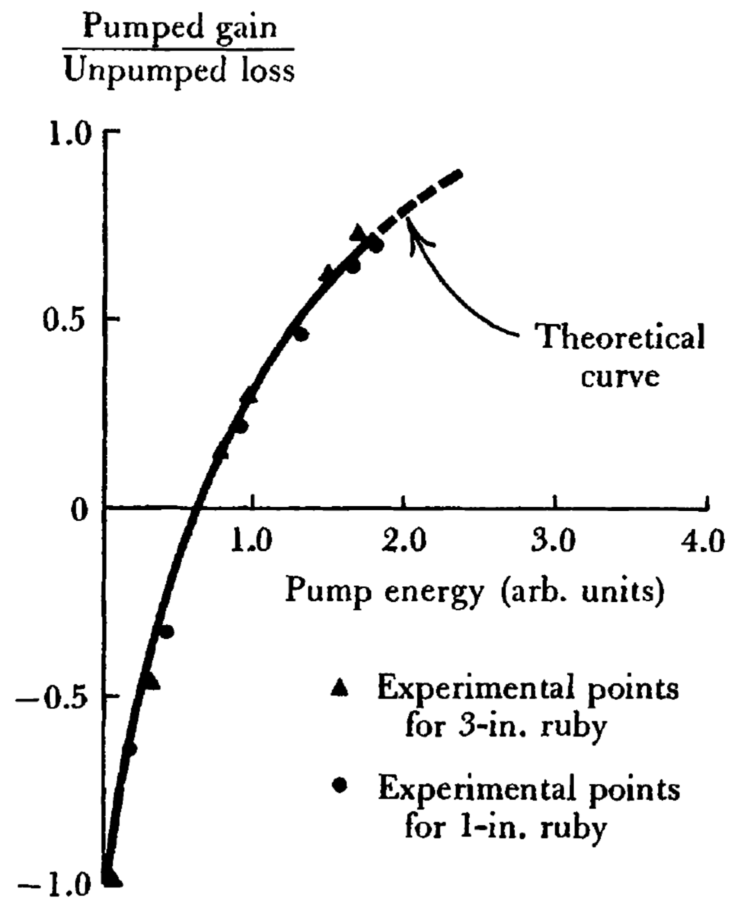
Integrate to get pop. Inv:

Square pulse:

$$\frac{\Delta N(t)}{N} = \frac{(W_p\tau - 1) - 2W_p\,\tau \cdot \exp\left[-\left(W_p\tau + 1\right)t/\tau\right]}{W_p\tau + 1}$$

If pump pulse duration is short ($T_p \ll \tau$),
and the pumping rate is high ($W_p\tau \gg 1$

$$\frac{\Delta N\left(T_p\right)}{N} \approx 1 - 2e^{-W_pT_p}$$



Simple model-agrees with experiment!

Summary

Steady state laser pumping and population inversion

4-level laser

$$\frac{N_3 - N_2}{N} \approx \frac{W_p \tau_{rad}}{1 + W_p \tau_{rad}}$$

3-level laser

$$\frac{N_2 - N_1}{N} \approx \frac{W_p \tau_{rad} - 1}{1 + W_p \tau_{rad}}$$

Difference between three and four-level systems, and why four-level systems are superior

Laser gain saturation

Upper-level laser, saturation behavior

$$\Delta N_{21} = \Delta N_0 \cdot \frac{1}{1 + W_{sig} \tau_{eff}}$$

Saturation intensity is **independent** of the pumping-rate "i.e. The signal intensity needed to reduce the pop. Inv. To half its initial value doesn't depend on the pumping rate"

Transient rate equations

Upper-level laser

$$\eta_p = \frac{N_2(t = T_p)}{R_{p0} T_p} = \frac{1 - e^{-T_p/\tau_2}}{T_p/\tau_2}$$

Short pulses are needed to obtain high pumping efficiency

Three-level laser

$$\frac{\Delta N(T_p)}{N} \approx 1 - 2e^{-W_p T_p}$$

These simple models give good agreement with reality

Contents

Wave propagation in an atomic medium

Maxwell's equations:
$\nabla x \boldsymbol{E} = -j\omega \boldsymbol{B}$
$\nabla x \boldsymbol{H} = \boldsymbol{J} + j\omega \boldsymbol{D}$

Constitutive relations:
$\boldsymbol{B} = \mu \boldsymbol{H}$
$\boldsymbol{J} = \sigma \boldsymbol{E}$
$\boldsymbol{D} = \epsilon \boldsymbol{E} + \boldsymbol{P}_{at} = \epsilon[1 + \boldsymbol{\chi}_{at}]\boldsymbol{E}$

Vector field of the form:
$$\boldsymbol{\epsilon}(\boldsymbol{r}, t) = \frac{1}{2}\left[\boldsymbol{E}(\boldsymbol{r})e^{j\omega t} + c.c\right]$$

Material parameters:
μ – magnetic permeability
σ – ohmic losses
ϵ – dielectric permittivity (not counting atomic transitions)
$\chi_{at}(\omega)$ – resonant susceptibility due to laser transitions

Assume a spatially uniform material ($\nabla \cdot E = 0$), and apply $\nabla \times$ to get the wave equation:

$$\left[\nabla^2 + \omega^2 \mu\epsilon \left(1 + \tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right)\right] \tilde{E}(x, y, z) = 0$$

Plane-wave approximation

Consider a plane wave,

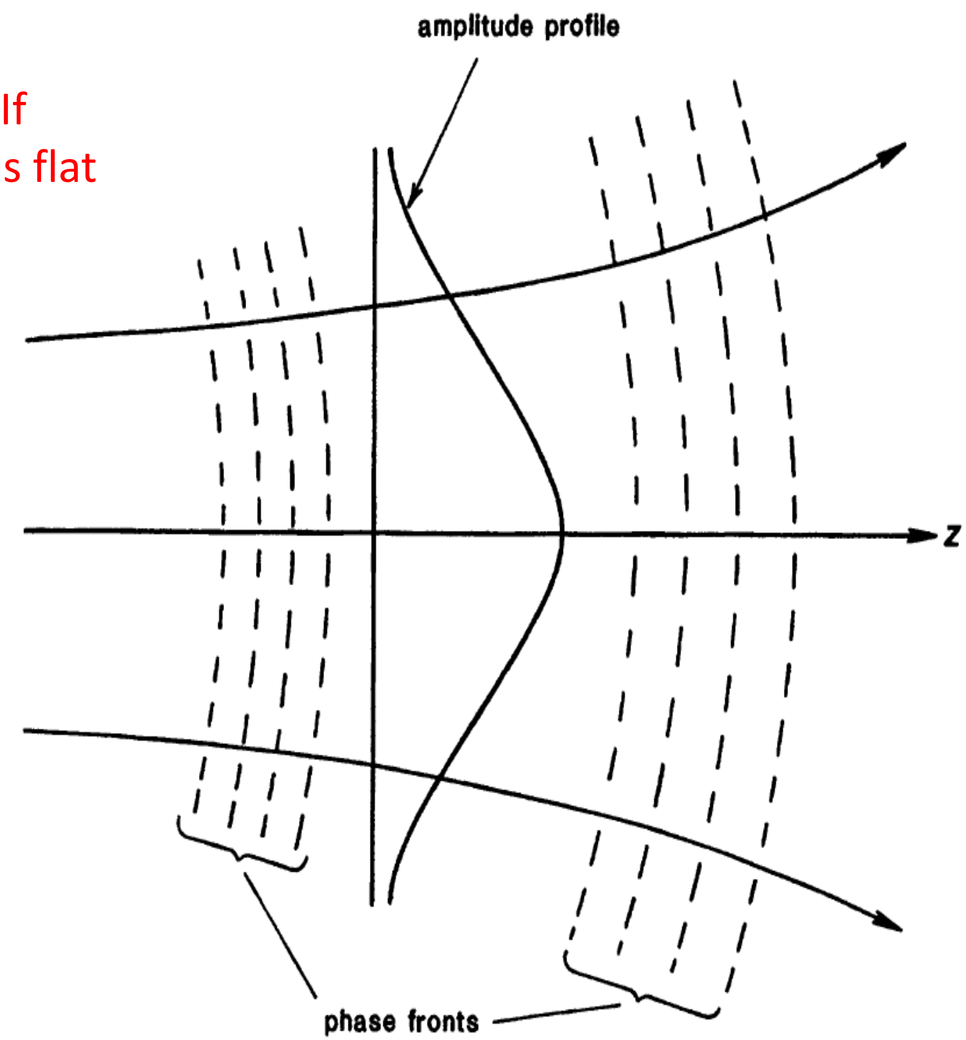
<- Ok approx. If wavefront is flat

$$\left|\frac{\partial^2 \tilde{E}}{\partial x^2}\right|, \left|\frac{\partial^2 \tilde{E}}{\partial y^2}\right| \ll \left|\frac{\partial^2 \tilde{E}}{\partial z^2}\right|$$

i.e. $\nabla^2 \rightarrow \frac{d^2}{dz^2}$

The equation reduces to:

$$\left[d_z^2 + \omega^2 \mu \epsilon \left(1 + \tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right)\right] \tilde{E}(z) = 0$$



Plane-wave approximation

Without losses:

First, no losses

$$[d_z^2 + \omega^2 \mu\epsilon]\tilde{E}(z) = 0,$$

Assume solutions on the form:
$\tilde{E}(z) = const \cdot e^{-\Gamma z}$

$$\Rightarrow [\Gamma^2 + \omega^2 \mu\epsilon]\tilde{E} = 0$$

The allowed values for Γ are,
$\Gamma = \pm j\omega\sqrt{\mu\epsilon} \equiv \pm j\beta$

With the solution,

$$\boldsymbol{\epsilon}(z,t) = \frac{1}{2}\left[E_+ e^{j(\omega t - \beta z)} + E_+^* e^{-j(\omega t - \beta z)}\right] + \frac{1}{2}\left[E_- e^{j(\omega t + \beta z)} + E_-^* e^{-j(\omega t + \beta z)}\right]$$

The free space propagation constant, β, may be written:

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Different beta from last chapter

Plane-wave approximation

Put losses back

With laser action and losses:

$$\left[d_z^2 + \beta^2\left(1 + \tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right)\right]\tilde{E}(z) = 0$$

Assuming $\tilde{E}(z) = const \cdot e^{-\Gamma z}$, as before:

$$\Gamma = j\beta\sqrt{1 + \tilde{\chi}'_{at}(\omega) + j\tilde{\chi}''_{at}(\omega) - j\sigma/\omega\epsilon}$$

Normally, $\tilde{\chi}_{at}(\omega), \frac{\sigma}{\omega\epsilon} \ll 1$:

Expand the sqrt

$$\sqrt{1+\delta} \approx 1 + \frac{\delta}{2}$$

$$\Gamma \approx j\beta\left[1 + \frac{1}{2}\chi'_{at}(\omega) + \frac{j}{2}\,\chi''_{at}(\omega) - \frac{j\sigma}{2\omega\epsilon}\right] =$$

$$\equiv j\beta + j\Delta\beta_m(\omega) - \alpha_m(\omega) + \alpha_0$$

Define four terms

The final solution becomes:

$$\boldsymbol{\epsilon}(z,t) = Re\tilde{E}_0 \exp[j\omega t - j[\beta + \Delta\beta_m(\omega)]z + [\alpha_m(\omega) - \alpha_0]z]$$

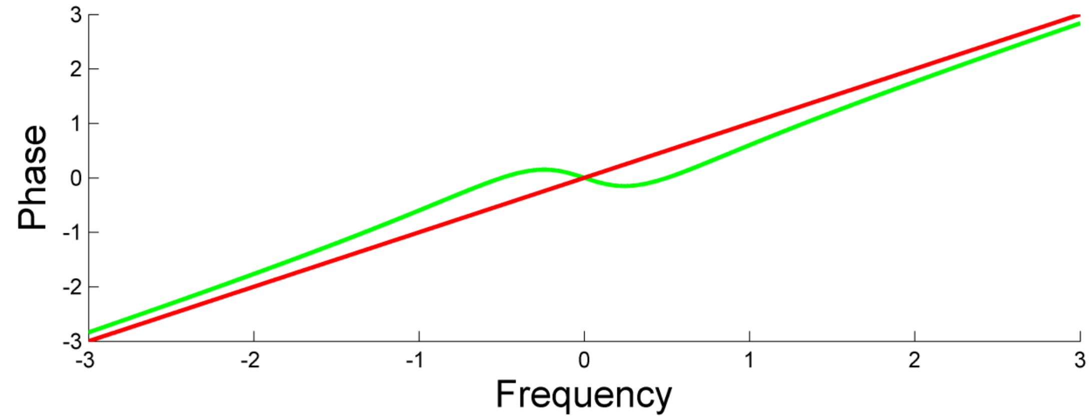
Propagation Factors

$$\boldsymbol{\epsilon}(z,t) = Re\tilde{E}_0 \exp[j\omega t - j[\beta + \Delta\beta_m(\omega)]z + [\alpha_m(\omega) - \alpha_0]z]$$

Phase-shift **loss/gain**

Linear phase-shift (Red)

β – Plane-wave propagation constant, fundamental phase variation,



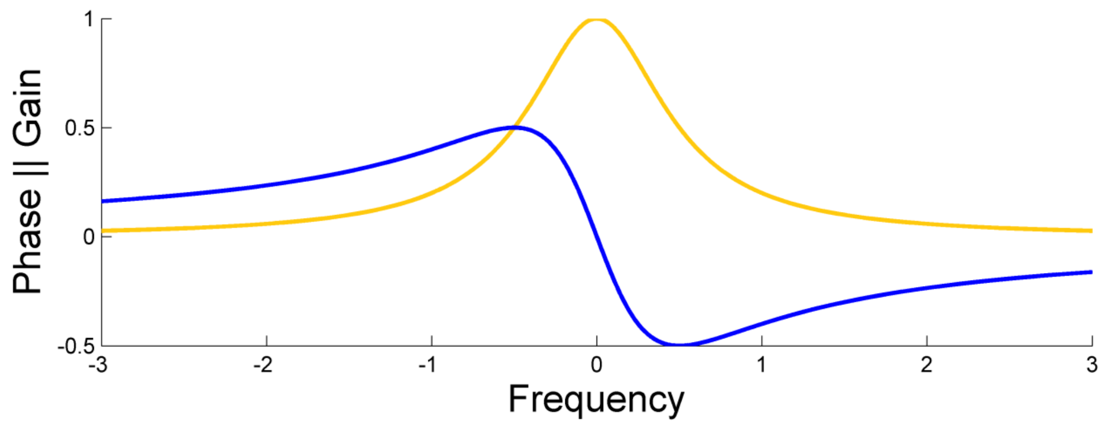
Nonlinear phase-shift (Blue)

$\Delta\beta_m(\omega)$ – Additional atomic phase-shift due to population inversion

→ Total phase-shift (Green)

Gain (Orange)

$\alpha_m(\omega)$ – Atomic gain or loss coefficient (due to transitions)



Background

α_0 – Ohmic background loss

Exceptions

Larger atomic gain or absorption effects

For expanding the sqrt

The results so far are based on the assumption that $\left|\tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right| \ll 1$, however, there are a few situations of interest where this doesn't hold:

- Absorption in metals and semiconductors, at frequencies **higher than the bandgap energy**, the effective conductivity, σ can become very large

Big sigma

- Absorption on **strong resonance lines** in metal vapor, the transitions are very strongly allowed giving a high absorption per unit length

Big chi

The paraxial wave equation

Want to handle **transverse variations**

The full wave-equation,

$$\left[\nabla^2 + \beta^2 \left(1 + \tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right)\right] \boldsymbol{E}(\boldsymbol{r}) = 0$$

New ansatz,

$$\tilde{E}(\boldsymbol{r}) \equiv \tilde{u}(\boldsymbol{r}) e^{-j\beta z}$$ change **const** -> **u(r)**

Insert into the wave equation,

$$\left[\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{\partial^2 \tilde{u}}{\partial z^2} - 2j\beta \frac{\partial \tilde{u}}{\partial z} - \beta^2 \tilde{u}\right] e^{-j\beta z} = 0$$

Assume that $\tilde{u}(\boldsymbol{r})$ changes slowly along the z-direction,

$$\left|\frac{\partial^2 \tilde{u}}{\partial z^2}\right| \ll \left|2\beta \frac{\partial \tilde{u}}{\partial z}\right|$$

and

$$\left|\frac{\partial^2 \tilde{u}}{\partial z^2}\right| \ll \left|\frac{\partial^2 \tilde{u}}{\partial x^2}\right|, \left|\frac{\partial^2 \tilde{u}}{\partial y^2}\right|$$

"Paraxial wave equation"

$$\Rightarrow \nabla_t^2 \tilde{u} - 2j\beta \frac{\partial \tilde{u}}{\partial z} + \beta^2 \left(\tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right) \tilde{u} = 0$$

Diffraction- and propagation effects

Re-write the equation

Rewrite:

$$\frac{\partial \tilde{u}(\boldsymbol{r})}{\partial z} = -\frac{j}{2\beta}\,\nabla_t^2 \tilde{u}(\boldsymbol{r}) - [\alpha_0 - \alpha_m + j\Delta\beta_m]\tilde{u}(\boldsymbol{r})$$

α_0, $\alpha_m(\omega)$, and $j\Delta\beta_m(\omega)$ are defined as before

Four terms, same as before

Two terms:

Diffraction, $-\frac{j}{2\beta}\,\nabla_t^2$

Ohmic and atomic gain/loss, $-[\alpha_0 - \alpha_m + j\Delta\beta_m]$

Separate terms –> **Independent to first order**, gain and phase-shift results are the same as for **plane waves**

Reproduce the **plane-wave result**

Laser amplification

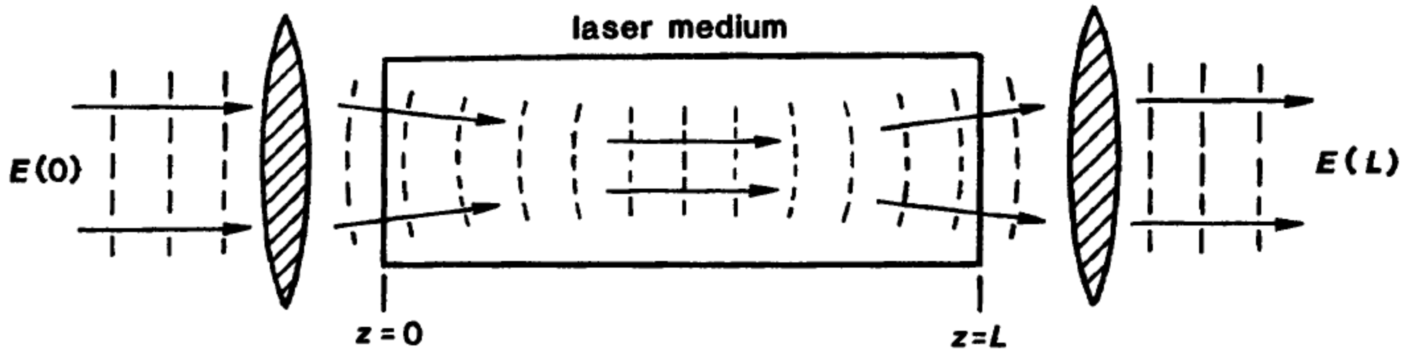


FIGURE 7.7
An elementary single-pass laser amplifier.

Calculate the gain

The laser gain after length L:

$$\tilde{g}(\omega) \equiv \frac{\tilde{E}(L)}{\tilde{E}(0)}$$

In terms of intensity,

from def. alpha

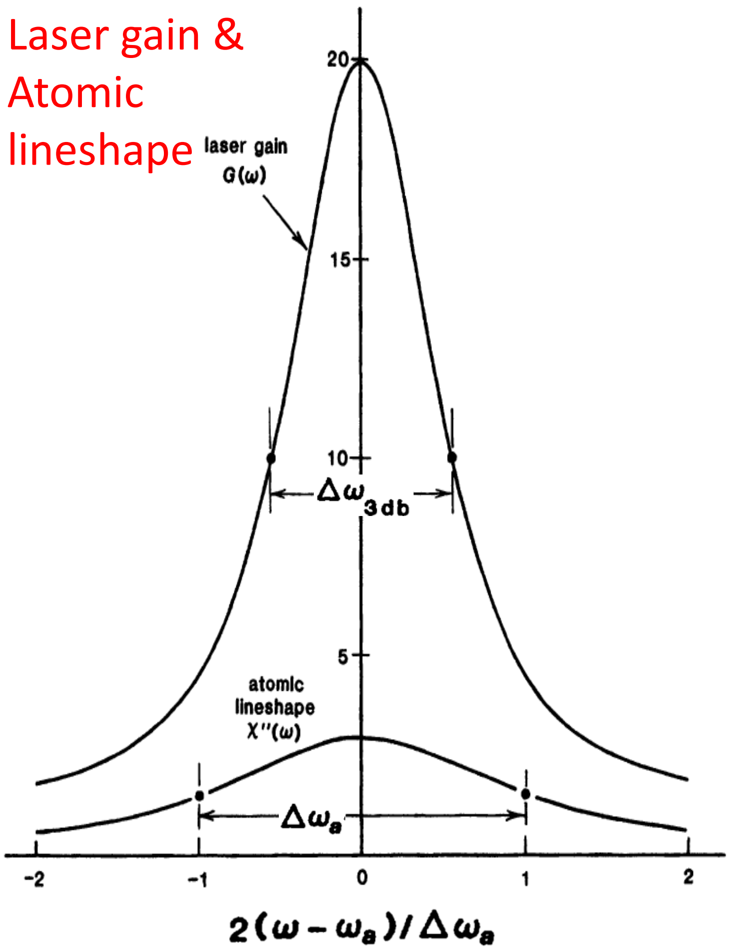
$$G(\omega) = \frac{I(L)}{I(0)} = \frac{\left|\tilde{E}(L)\right|^2}{\left|\tilde{E}(0)\right|^2} = \exp[2\alpha_m(\omega)L - 2\alpha_0 L] = \exp[\beta\chi''(\omega)L - \frac{\sigma}{\epsilon c}L]$$

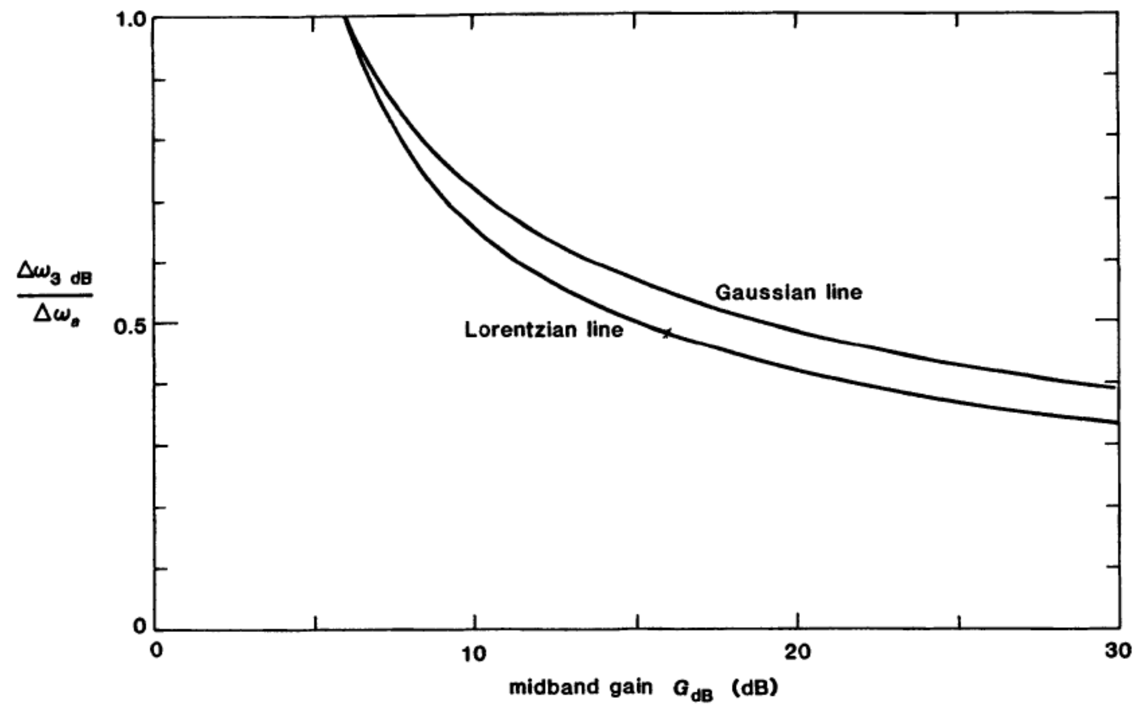
For most lasers, $\alpha_0 \ll \alpha_m \Rightarrow$ $$G(\omega) \approx \exp(\beta\chi''(\omega)L)$$ exponential dependence on chi

Gain narrowing

The gain, $G(\omega) \sim \exp[\chi''(\omega)] \rightarrow$ Narrower frequency dependence, i.e. "Gain Narrowing"

Laser gain & Atomic lineshape



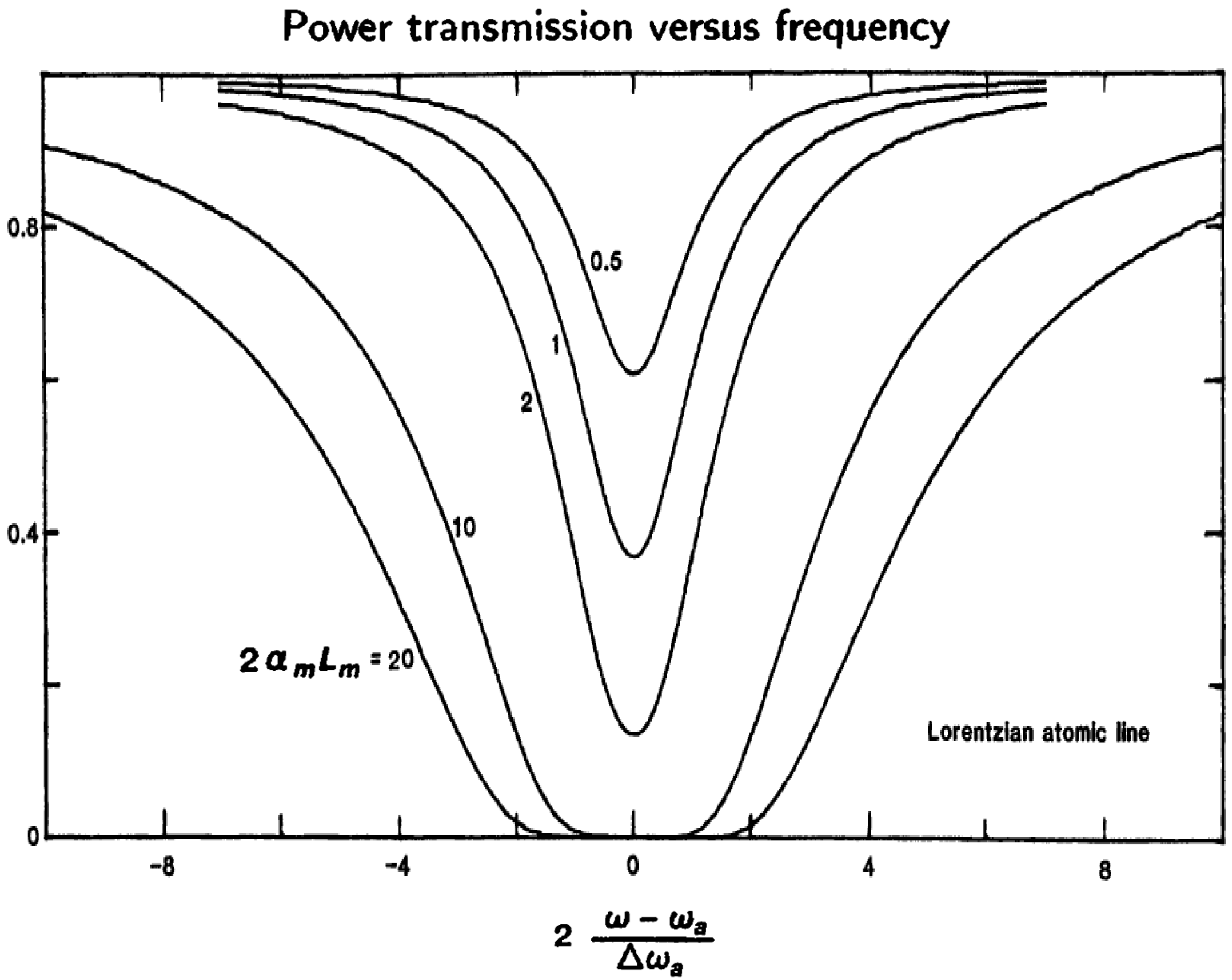


FWHM bandwidth is lowered by 30-40%

Absorption broadening

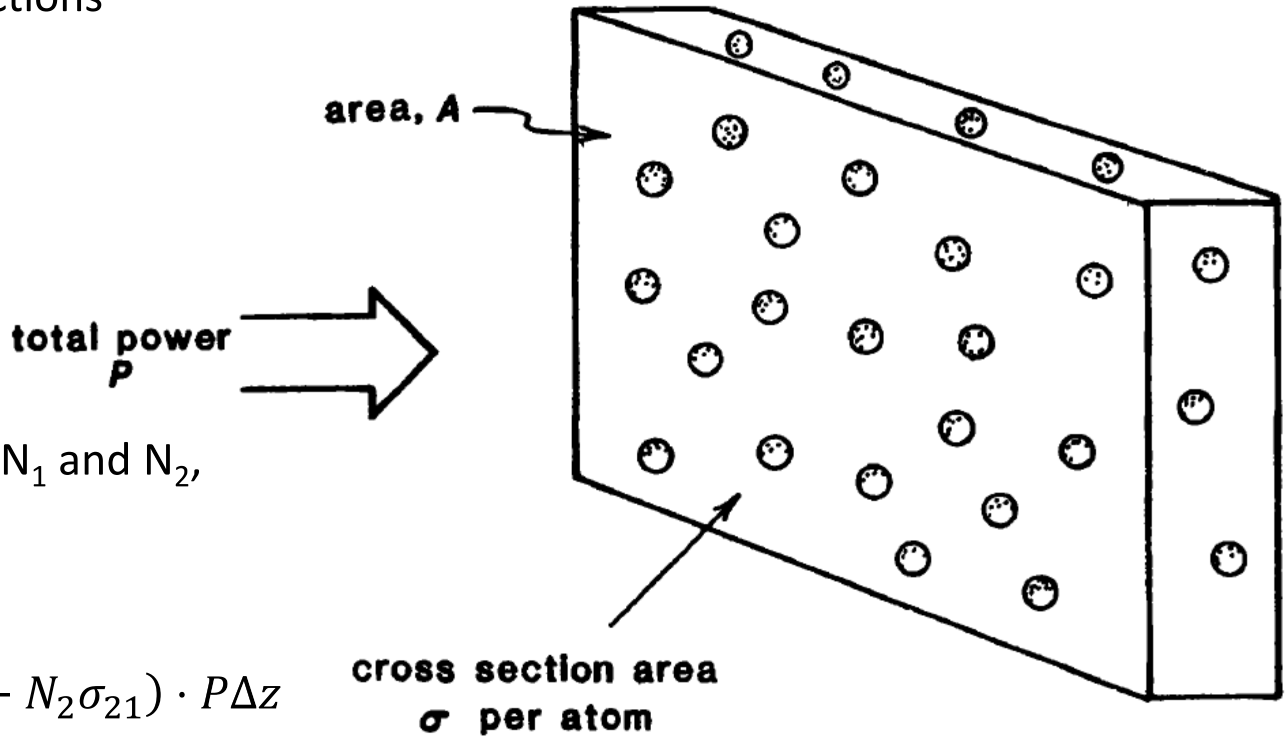
Absorbing media have opposite sign of $\chi(\omega)$
→ "**Absorption broadening**"

Uninverted population
-> absorption broadening



Stimulated transition cross-sections

For a black-body:
$\Delta P_{abs} = \sigma \cdot I$



Thin slab, atomic densities N_1 and N_2, cross-sections σ_{12} and σ_{21}

Net absorbed power:
$$\Delta P_{abs} = (N_1\sigma_{12} - N_2\sigma_{21}) \cdot P\Delta z$$

The cross-sections are related,
$$g_1\sigma_{12} = g_2\sigma 21$$

Convert power → intensity:
$$\frac{1}{I}\frac{dI}{dz} = -\Delta N_{12}\sigma_{21}$$

Previously:
$$\frac{1}{I}\frac{dI}{dz} = -2\alpha_m(\omega)$$

Loss or gain can be calculated from cross-sections and atomic density

$$\Rightarrow 2\alpha_m(\omega) = \Delta N_{12}\sigma_{21}(\omega)$$

Cross-section formula

From previous page: $2\alpha_m(\omega) = \Delta N_{12}\sigma_{21}(\omega)$

From definition of α: $\alpha_m(\omega) = \frac{\pi\chi''(\omega)}{\lambda}$

$$\Rightarrow 2\alpha_m(\omega) = \Delta N\sigma(\omega) = \frac{2\pi}{\lambda}\chi''(\omega)$$

The cross-section is a function of wavelength, transition rate and lineshape

Using the full expression for χ:

$$\sigma(\omega_a) = \frac{3^*}{2\pi}\frac{\gamma_{rad}}{\Delta\omega_a}\lambda^2 \cdot \frac{1}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2}$$

Cross-section estimation: Estimate the max cross-section

Assume, $\Delta\omega_a \equiv \gamma_{rad.}$ (purely radiative transmission), $3 = 3^*$, (all atoms aligned)

$$\Rightarrow \sigma_{max} = \frac{3\lambda^2}{2\pi} \approx \frac{\lambda^2}{2} \approx 10^{-9} cm^2$$

The cross section is far greater than the area of an atom, due to its **internal resonance**.

Real cross-sections

TABLE 7.1
Typical Laser Transition Cross Sections

Laser system	*Transition cross section σ*
Gas lasers in the visible and near IR	10^{-11} to $10^{-13} cm^2$
Low-pressure CO_2 laser (10.6μm)	$3 \times 10^{-18} cm^2$
Organic dye laser (Rhodamine 6G)	1 to $2 \times 10^{-16} cm^2$
Nd^{3+} ion in Nd:YAG	$4.6 \times 10^{-19} cm^2$
Nd^{3+} ion in Nd:glass	$3 \times 10^{-20} cm^2$
Cr^{3+} ion in ruby	$2 \times 10^{-20} cm^2$

$\frac{\lambda^2}{2} >$ Real cross-sections $\gg$ Atom area

Allowed transitions in gas and non-allowed in solids -> $\tau_{gas} \ll \tau_{solid}$

Population difference saturation

Gain saturation,

In a medium,

$$\frac{dI}{dz} = \pm 2\alpha_m I = \pm \Delta N \sigma I$$

As shown previously, ΔN saturates according to:

$$\Delta N = \Delta N_0 \cdot \frac{1}{1 + W\tau_{eff}} = \Delta N_0 \cdot \frac{1}{1 + I/I_{sat}}$$

N_0 - unsaturated or small signal gain

I_{sat} - saturation intensity

As ΔN saturates, the **gain saturates**

Stimulated-transition probability,

From before, $\Delta P_{abs} = (N_1\sigma_{12} - N_2\sigma_{21}) \cdot P\Delta z$

From rate equation analysis,$\Delta P_{abs} = (W_{12}N_1 - W_{21}N_2)A\Delta z\hbar\omega_a$

$$\Rightarrow \mathrm{W} = \frac{\sigma \mathrm{I}}{\hbar\omega}$$

i.e. the cross-section, the **intensity** and the **stimulated transition probability** are **interdependent**

Population difference saturation

At the line center:

From previous slide,

$$2\alpha_m(\omega, I) = \frac{\Delta N_0}{1 + I/I_{sat}} = \frac{\Delta N_0 \sigma}{1 + \left(\sigma \tau_{eff}/\hbar\omega\right) I}$$

Re-arrange prev expressions

$$\rightarrow I_{sat} \equiv \frac{\hbar\omega}{\sigma \tau_{eff}}$$

i.e. $I = I_{sat}$: one incident photon within the cross-section per recovery time

Off center:

The expression is modified by the lineshape,

$$2\alpha_m(\omega, I) = \frac{\Delta N_0 \sigma}{1 + \frac{I}{I_{sat}} \cdot \frac{1}{1 + y^2}}$$

Where y is the normalized frequency detuning,

$$y = 2(\omega - \omega_0)/\Delta\omega$$

Frequency dependence due to, **transition lineshape** and **saturation behavior**

Practical values

From previous slide,

$$I_{sat} \equiv \frac{\hbar\omega}{\sigma\tau_{eff}}$$

I-sat varies a lot between materials

I_{sat} is an important parameter since little only little gain will be obtained once the intensity approaches this level.

Laser type:	$\hbar\omega$	σ	τ_{eff}	I_{sat}
Gas	$10^{-19}\,J$	$10^{\mathbf{-13}}\,cm^2$	$10^{\mathbf{-6}}\,s$	$1\,\boldsymbol{W}/cm^2$
Solid-state	$10^{-19}\,J$	$10^{\mathbf{-19}}\,cm^2$	$10^{\mathbf{-3}}\,s$	$1\,\boldsymbol{kW}/cm^2$

I_{sat} is **independent** of the **pumping intensity**, since neither σ nor τ_{eff} are intensity dependent.

However, harder pumping does increase the **small-signal gain**

Saturation in laser amplifiers

Calculate the saturation

As a signal passes through an amplifier, it grows exponentially with distance until the intensity approaches I_{sat}

For a single pass amplifier:

$$\frac{1}{I(z)}\frac{dI}{dz} = 2\alpha_m(I) = \frac{2\alpha_{m0}}{1 + I(z)/I_{sat}}$$

Integrating,

$$\int_{I=I_{in}}^{I=I_{out}} \left[\frac{1}{I} + \frac{1}{I_{sat}}\right] dI = 2\alpha_{m0} \int_{z=0}^{z=L} dz$$

gives:

$$\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}} = 2\alpha_{m0}L = \ln(G_0)$$

where, G_0 is the small-signal gain

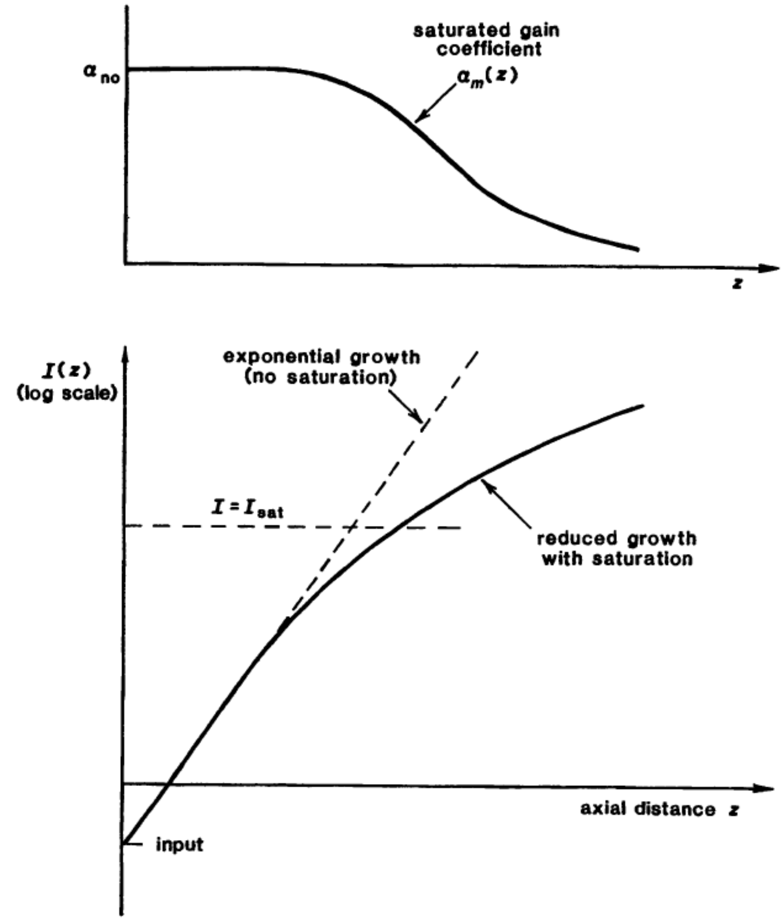


FIGURE 7.12
Gain saturation as a function of distance along a single-pass laser amplifier.

Saturation in laser amplifiers

The total gain,

$$G \equiv \frac{I_{out}}{I_{in}} = G_0 \cdot \exp(-\frac{I_{out} - I_{in}}{I_{sat}})$$

Is the unsaturated gain, reduced by a factor exponentially dependent on $I_{out} - I_{in}$

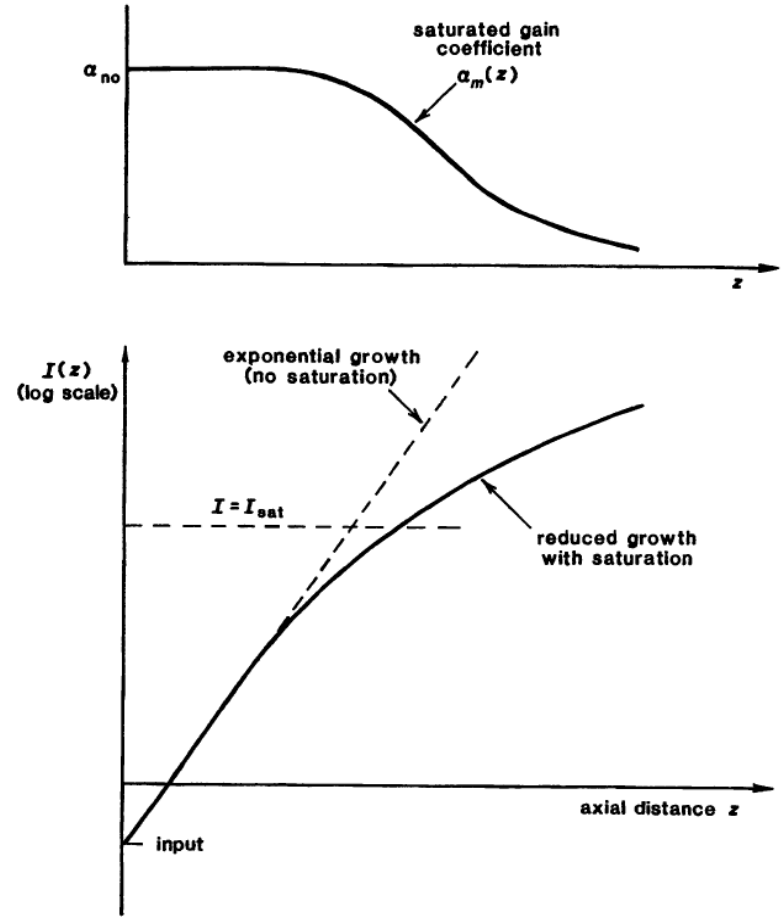
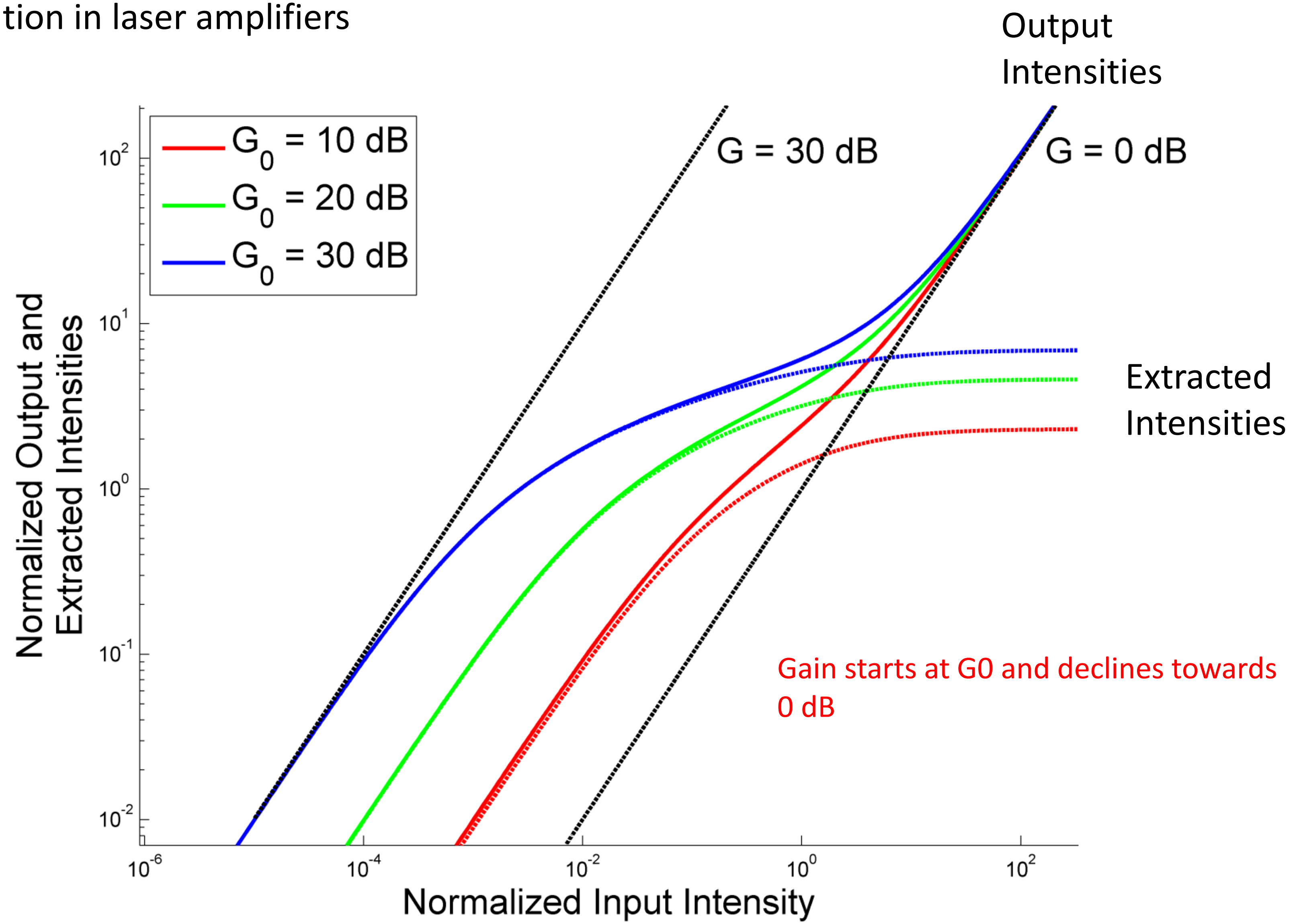


FIGURE 7.12
Gain saturation as a function of distance along a single-pass laser amplifier.

Saturation in laser amplifiers



Plotting the normalized output intensity vs the normalized input intensity, it is clear that the **transmission tends to unity** as the **input intensity is increased** -The amplifier becomes **transparent**.

Power-extraction

Define the extracted intensity,

Calculate the extracted power

$$I_{extr} \equiv I_{out} - I_{in} = \ln\left(\frac{G_0}{G}\right) \cdot I_{sat}$$

As the amplifier saturates, $G \rightarrow 1$,

$$I_{avail} = \lim_{G \rightarrow 1} \ln\left(\frac{G_0}{G}\right) \cdot I_{sat} = \ln(G_0) \cdot I_{sat}$$

Using $\ln(G_0) = 2\alpha_{m0}L$:

$$I_{avail} = 2\alpha_{m0}L \cdot I_{sat} = (\Delta N_0 \sigma L) \cdot \left(\frac{\hbar\omega}{\sigma\tau_{eff}}\right)$$

And re-writing:

$$\frac{I_{avail}}{L} \equiv \frac{P_{avail}}{V} = \frac{\Delta N_0 \hbar\omega}{\tau_{eff}}$$

i.e. the maximum **available power** is the **total inversion energy**, $\Delta N_0 \hbar\omega$, once every **recovery time**, τ_{eff}

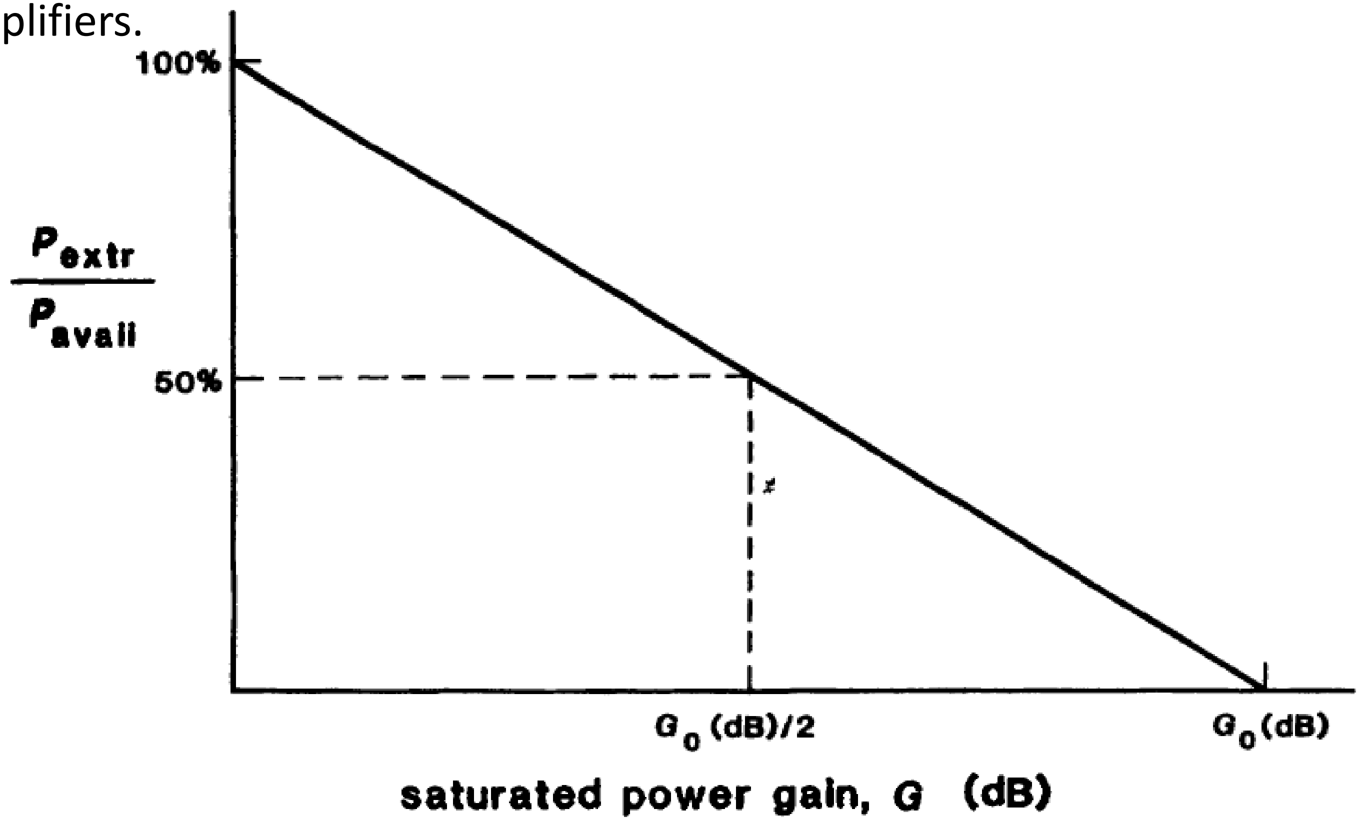
Power-extraction efficiency

Full power extraction requires complete saturation of the amplifier which gives low small-signal gain.

Define the extraction efficiency,

$$\eta_{extr} \equiv \frac{I_{extr}}{I_{avail}} = \frac{\ln(G_0) - \ln(G)}{\ln(G_0)} = 1 - \frac{G_{dB}}{G_{0,dB}}$$

i.e. there's a tradeoff between **effiency** and **small-signal gain** for single-pass amplifiers.



Summary

Wave propagation in atomic media

Plane-wave approximation,

$$\left[d_z^2 + \beta^2\left(1 + \tilde{\chi}_{at} - \frac{j\sigma}{\omega\epsilon}\right)\right]\tilde{E}(z) = 0$$

The paraxial wave equation,

$$\frac{\partial\tilde{u}(\boldsymbol{r})}{\partial z} = -\frac{j}{2\beta}\nabla_t^2\tilde{u}(\boldsymbol{r}) - [\alpha_0 - \alpha_m + j\Delta\beta_m]\tilde{u}(\boldsymbol{r})$$

β – Plane-wave propagation constant, fundamental phase variation,

$\Delta\beta_m(\omega)$ – Additional atomic phase-shift due to population inversion

$\alpha_m(\omega)$ – Atomic gain or loss coefficient (due to transitions)

α_0 – Ohmic background loss

Diffraction and gain are separate to first order

Single-pass laser amplification

Gain narrowing,

$$G(\omega) \sim \exp[\chi''(\omega)]$$

30-40% lower FWHM bandwidth

Transition cross-sections,

$$2\alpha_m(\omega) = \Delta N_{12}\sigma_{21}(\omega)$$

Loss/gain can be calculated from atomic density and cross-sections

Gain saturation,

$$G = G_0 \cdot \exp\left(-\frac{I_{out} - I_{in}}{I_{sat}}\right)$$

Low signal gain is saturated by a factor exponentialy dependent on the intensity difference

Power extraction,

$$\eta_{extr} = 1 - \frac{G_{dB}}{G_{0,dB}}$$

There’s a tradeoff between efficiency and small-signal gain