Classical Oscillator model and electric dipole transitions in real atoms

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Laser Physics SK3410
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Somewhere in Alba Nova

Outline

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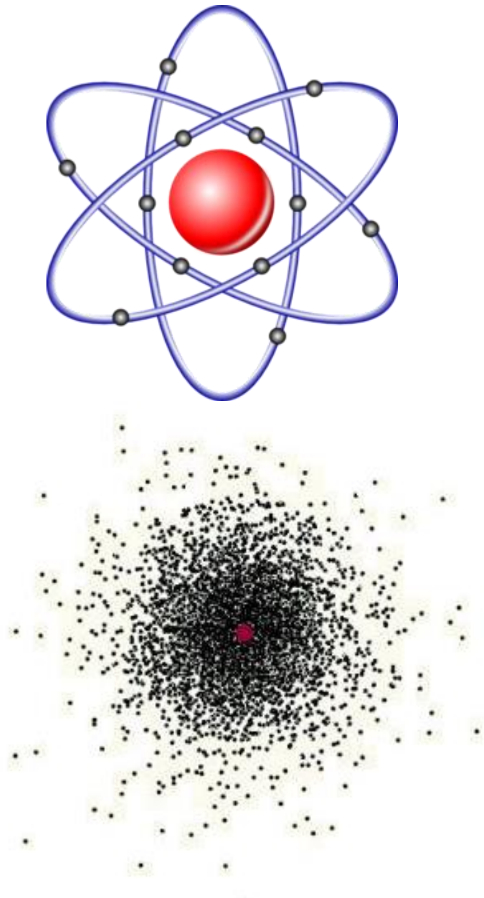
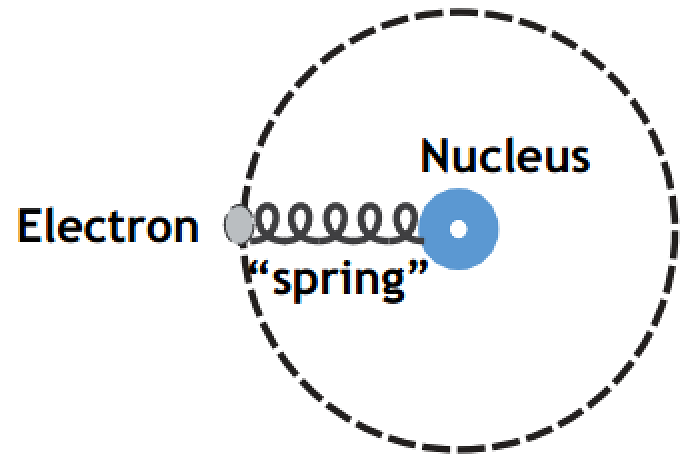
Based on “Lasers” by A. E. Siegman, 1986

What is CEO?

CEO – chief executive officer

CEO - Classical electron oscillator





Electronic models for real atoms

Classical electron oscillator model

Short CEO description

$$m\frac{d^2x(t)}{dt^2} = -Kx(t) - e\mathcal{E}_x(t),$$

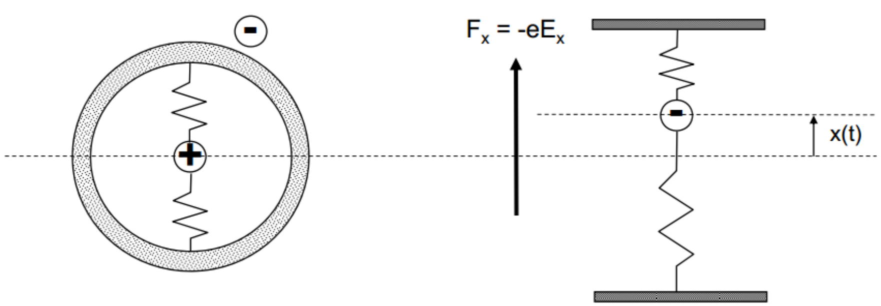
$Kx(t)$ Restoring force

$\mathcal{E}_x(t)$ External electrical field

$$\frac{d^2x(t)}{dt^2} + \omega_a^2 x(t) = -(e/m)\mathcal{E}_x(t),$$

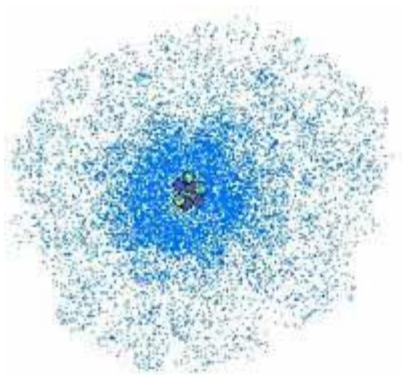
$$\omega_a^2 \equiv K/m.$$

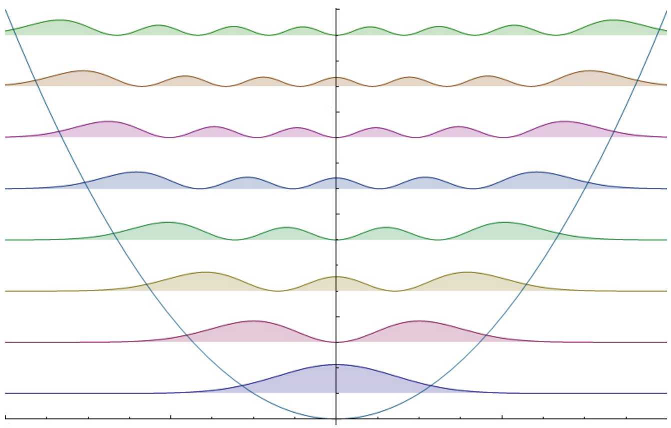
Resonance frequency



Mathematics (Siegman book p. 81)

But wait! We are in quantum world!





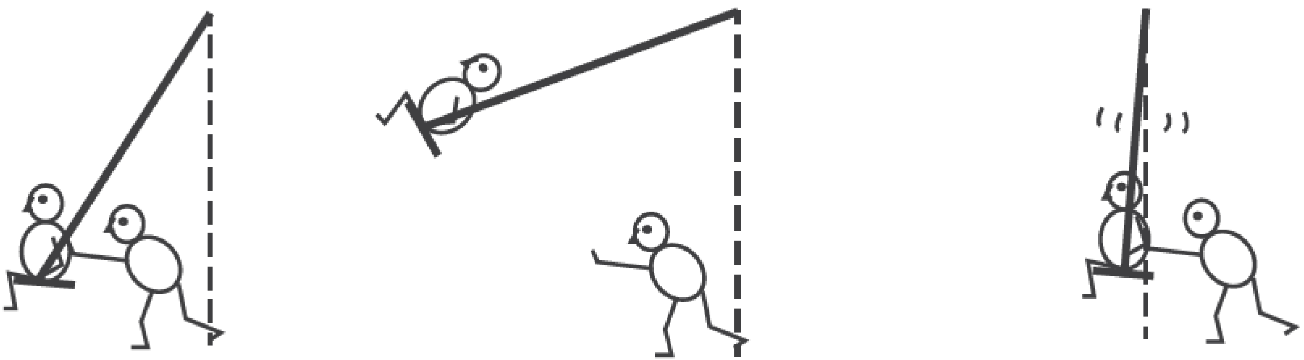
$$\omega_a^2 \equiv K/m.$$

$$\omega_{21} \equiv (E_2 - E_1)/\hbar$$

Real atoms

What is resonance frequency?

Almost real life illustration



Low frequency

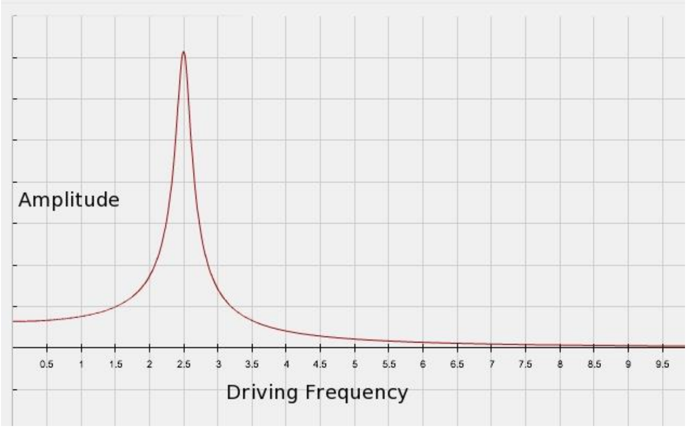
medium amplitude

At resonance

large amplitude

High frequency

vanishing amplitude



No references- random google search pictures.

Energy damping #1

$$\frac{d^2x(t)}{dt^2} + \omega_a^2 x(t) = -(e/m)\mathcal{E}_x(t),$$

Ideal life

↓

Real life

$$\frac{d^2x(t)}{dt^2} + \gamma\frac{dx(t)}{dt} + \omega_a^2 x(t) = -\frac{e}{m}\mathcal{E}_x(t),$$

Siegman book p. 83

Energy damping #2

1 $$\frac{d^2x(t)}{dt^2} + \gamma\frac{dx(t)}{dt} + \omega_a^2 x(t) = -\frac{e}{m}\mathcal{E}_x(t),$$ No electric field

2 $$x(t) = x(t_0)\exp[-(\gamma/2)(t-t_0) + j\omega_a'(t-t_0)],$$

Errata – displacement is in real part

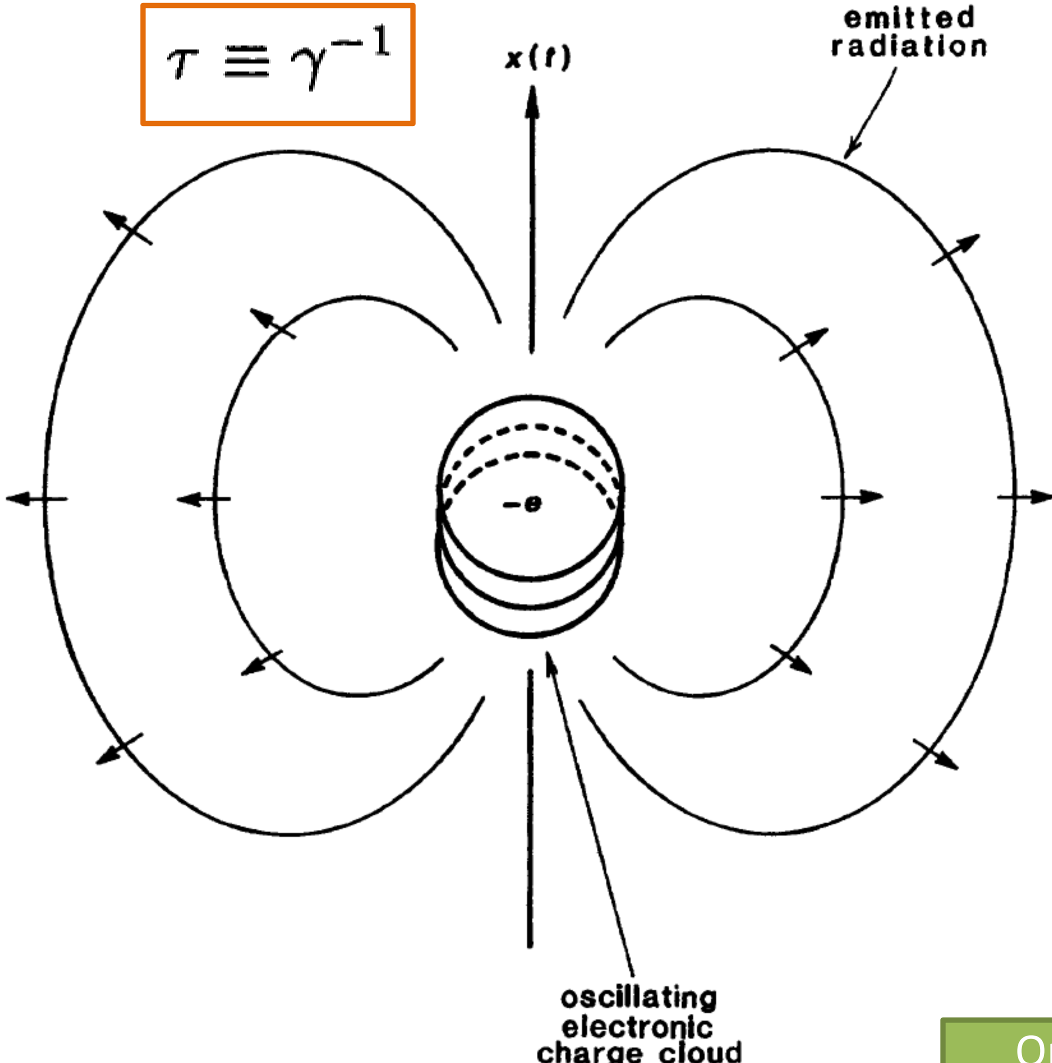
3 $$\omega_a' \equiv \sqrt{\omega_a^2 - (\gamma/2)^2}.$$ exact resonance frequency

4 $$U_a(t) = \frac{1}{2}Kx^2(t) + \frac{1}{2}mv_x^2(t) = U_a(t_0)e^{-\gamma(t-t_0)} \equiv U_a(t_0)e^{-(t-t_0)/\tau}.$$

Siegman book p. 82-83

Emission of electromagnetic radiation

$$\tau \equiv \gamma^{-1}$$

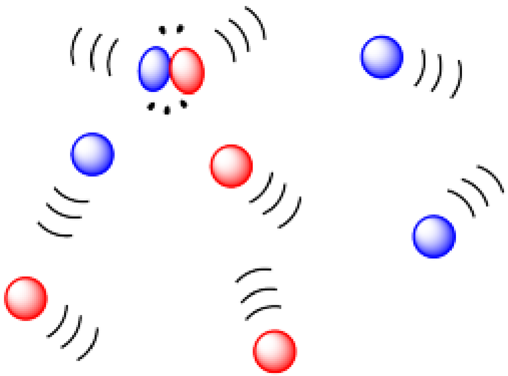


Only in movies…

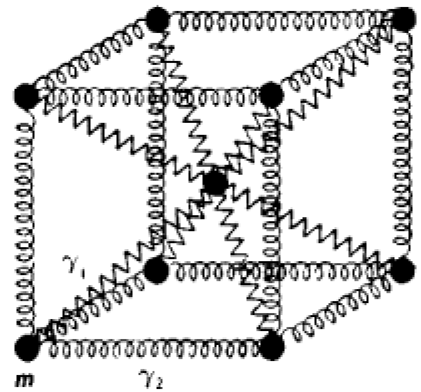
Siegman book p. 83

Real life- aka non "radiative" mechanisms

Collisions with other atoms



Lattice vibrations



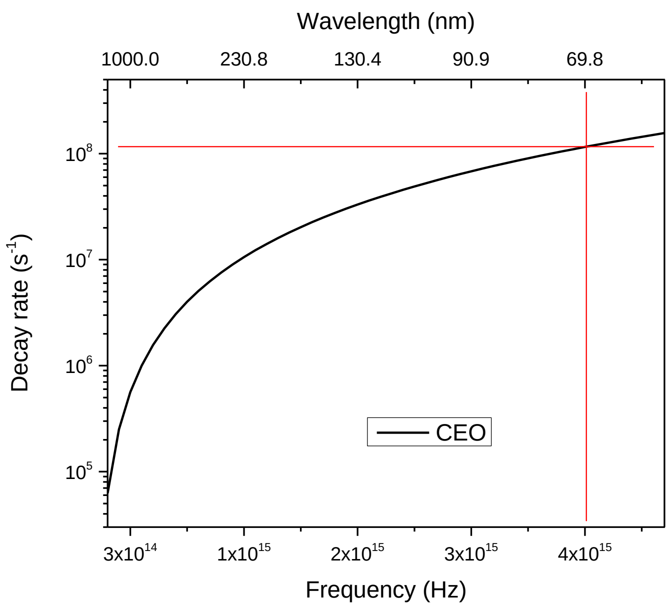
Internal charge cloud oscillation within atom

$$\gamma \equiv \left| \frac{1}{U_a} \frac{dU_a}{dt} \right| = \gamma_{\text{rad}} + \gamma_{\text{nr}}.$$

Errata – absolute values

Siegman book p. 83-84

Radiative decay rates



$$\gamma_{\mathrm{rad,ceo}} = \frac{e^2\omega_a^2}{6\pi\epsilon m c^3}.$$

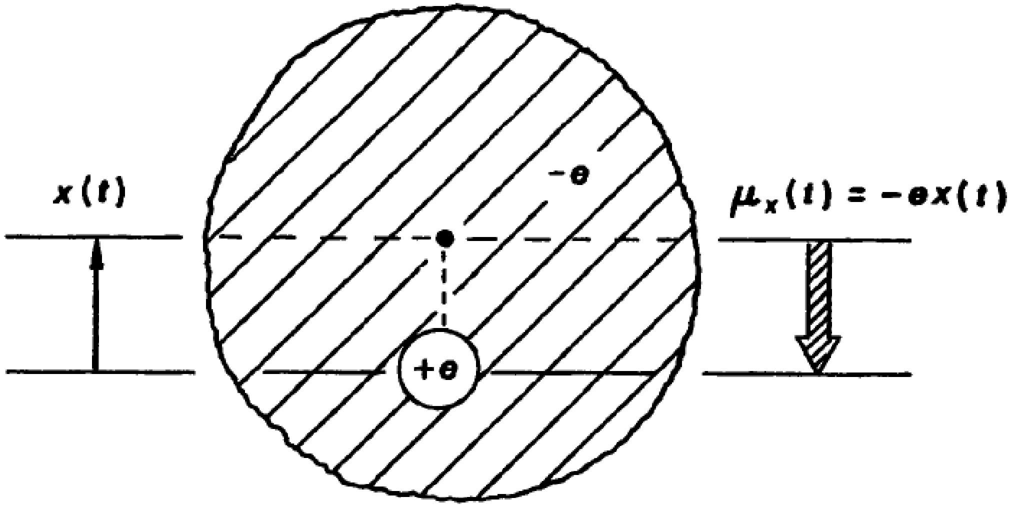
Parameter	Value
e	1.602e-19 (C)
ε	8.85e-12 (F/m)
m	9.11e-31 (kg)
c	3e8 (m/s)

This classical oscillator radiative decay rate has a value $\gamma_{\mathrm{rad,ceo}} \approx 10^8$ sec^{-1} for a visible frequency oscillator, compared to an oscillation frequency of $\omega_a \approx 4 \times 10^{15}$ sec^{-1}.

Siegman book p. 84

“UV vision”

Microscopic dipole moments



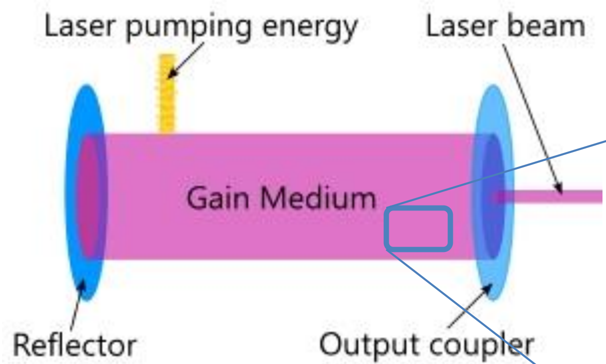
$$\mu_x(t) = [\text{charge}] \times [\text{displacement}] = -ex(t)$$

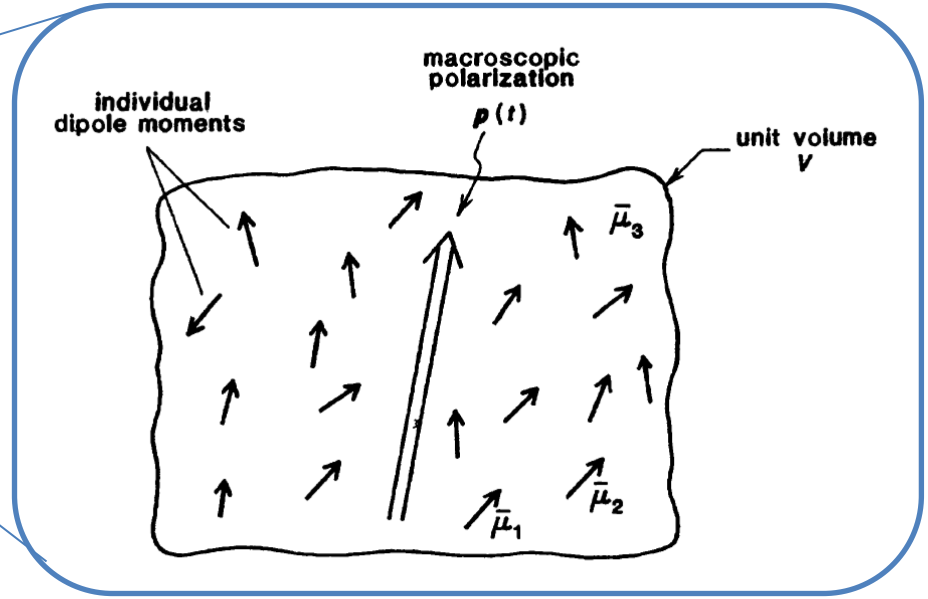
microscopic dipole moment

$$x(t) = x(t_0)\exp[-(\gamma/2)(t-t_0) + j\omega_a'(t-t_0)],$$

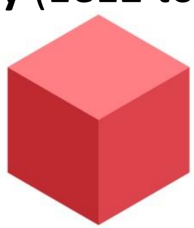
Siegman book p. 84-85

Macroscopic electric polarization





Medium density (1e12 to 1e22) in cm^3



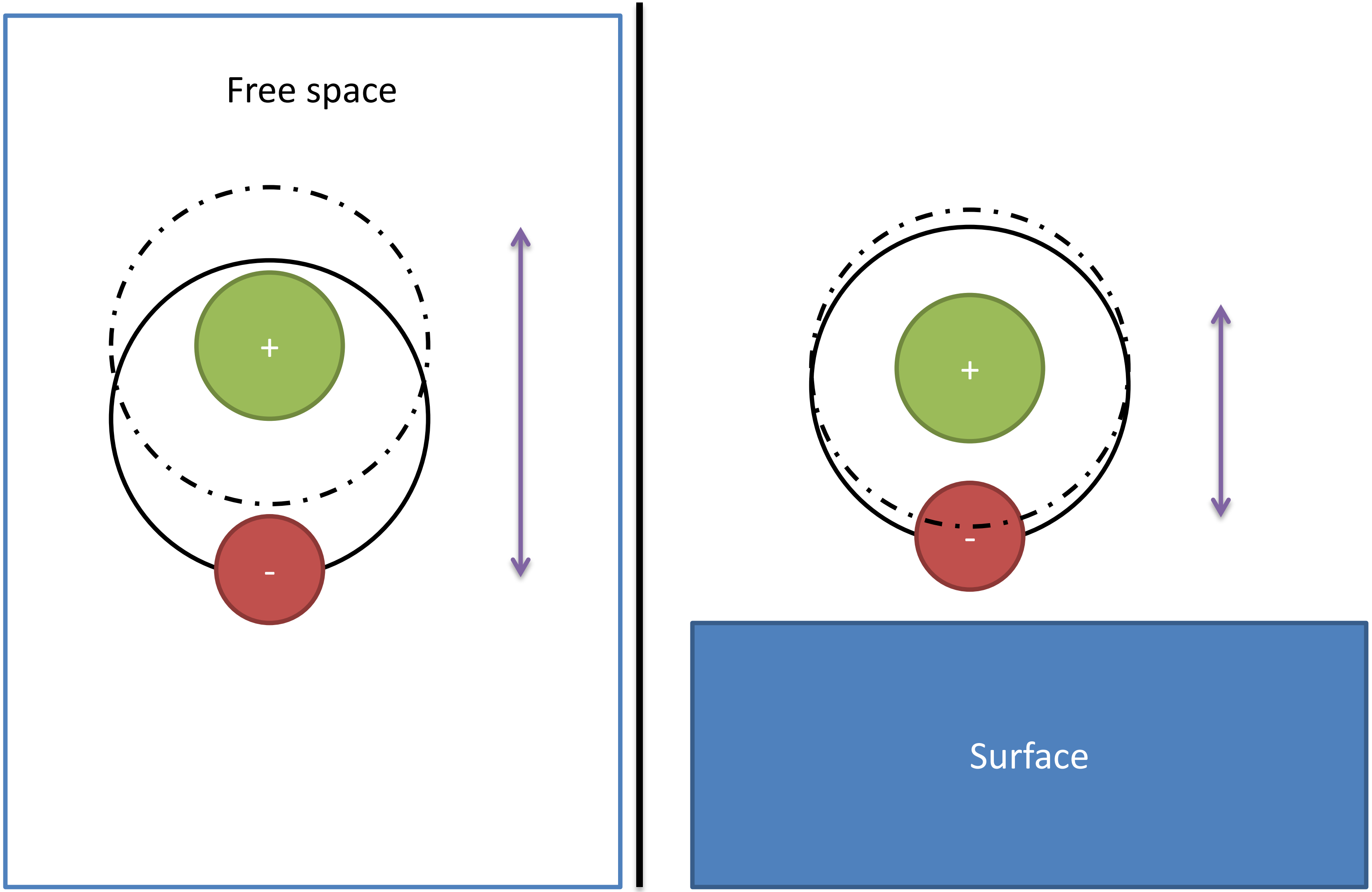
$$p_x(r,t) \equiv V^{-1} \sum_{i=1}^{NV} \mu_{xi}(t).$$

2 waves 500 nm
(2*(5e-5 cm))^3 = 1 e-12 cm^3
With density (1e18) we get 1e6 CEO’s

Macroscopic dipole moment

Siegman book p. 85-86

Classical world illustration



Siegman book p. 87

Coherent dipole oscillations #1

$$\frac{d^2\mu_x(t)}{dt^2} + \gamma\frac{d\mu_x(t)}{dt} + \omega_a^2\mu_x(t) = (e^2/m)\mathcal{E}_x(t)$$

x electron chage

$$\mu_x(t) = [\text{charge}] \times [\text{displacement}] = -ex(t)$$

+ microscopic dipole

$$\mu_x(t) = \mu_{x0}\,\exp\left[-(\gamma/2)(t-t_0) + j\omega_a(t-t_0) + j\phi_0\right],$$

Siegman book p. 89-90

Coherent dipole oscillations #2

$$\mu_{x,\text{tot}}(t) = \sum_{i=1}^{NV} \mu_{x,i}(t) = NV\mu_x(t) \quad \left\{ \begin{array}{l} \text{all dipoles} \\ \text{oscillating} \\ \text{in phase,} \end{array} \right.$$

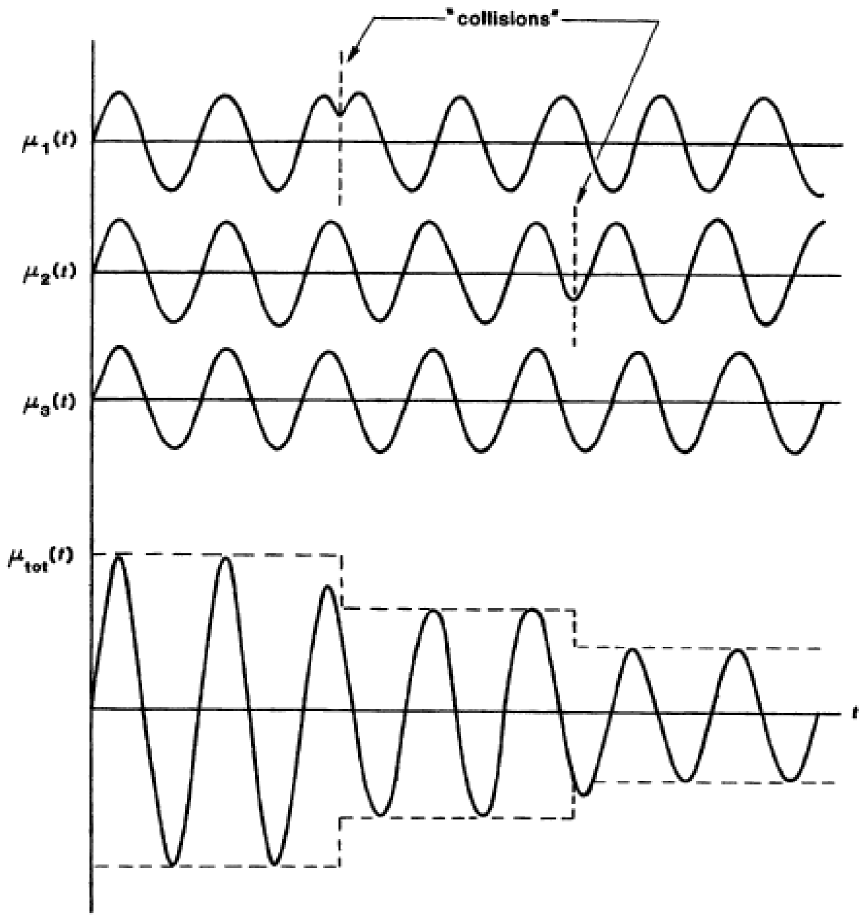
$$p_x(t) = \mu_{x,\text{tot}}(t)/V$$

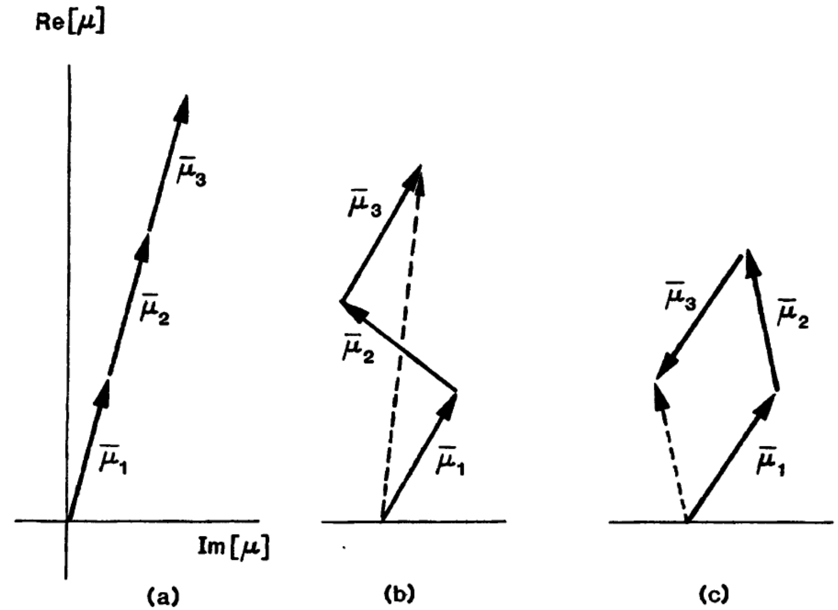
Unfortunately this is fantasy world

$$p_x(t) = N\mu_{x0}\, \exp[[-(\gamma/2) + j\omega_a](t - t_0) + j\phi_0] \quad \left\{ \begin{array}{l} \text{all dipoles} \\ \text{oscillating} \\ \text{in phase.} \end{array} \right.$$

Siegman book p. 89-90

Random collisions



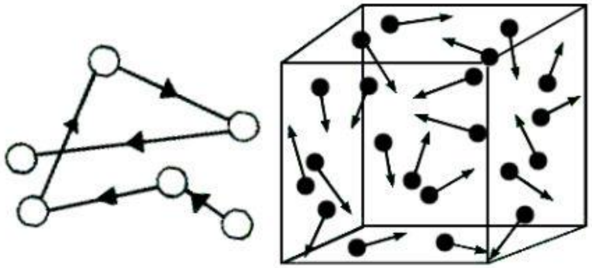


$$\langle \mu_{x,\text{tot}}(t) \rangle = 0 \qquad \text{(randomly phased dipoles),}$$

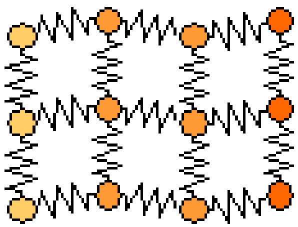
$$\langle \mu_{x,\text{tot}}^2(t) \rangle^{1/2} = (NV)^{1/2}\,|\mu_x(t)| \qquad \text{(randomly phased dipoles),}$$

Siegman book p. 91-92

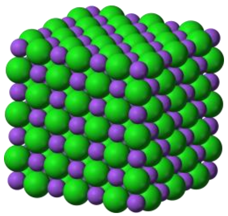
Dephasing mechanisms



Brownian motion (atoms, or ions, or molecules). Even if coalition is elastic, no energy is transferred, electronic oscillation phase will be changed.



Solid state lasers, the quantum energy-level spacing and ω are affected by neighboring atoms. Thermal vibrations of lattice, will modulate them.



In materials there atoms are sufficiently dense, oscillating dipole may spread out to, and be felt by, other neighboring laser atoms. Even week coupling tends to randomize overall response. (1e-13 sec)

Siegman book p. 93

The dephasing time

Macroscopic polarization initial state

$$p_x(t_0) = N_0 \mu_{x0}.$$

N(t)- dipoles that have not suffered at least one coalition

$$p_x(t) = N(t)\mu_x(t) = N(t)\mu_{x0} \cos\omega_a t,.$$

Errata

Suppose that collisions occur at a random rate

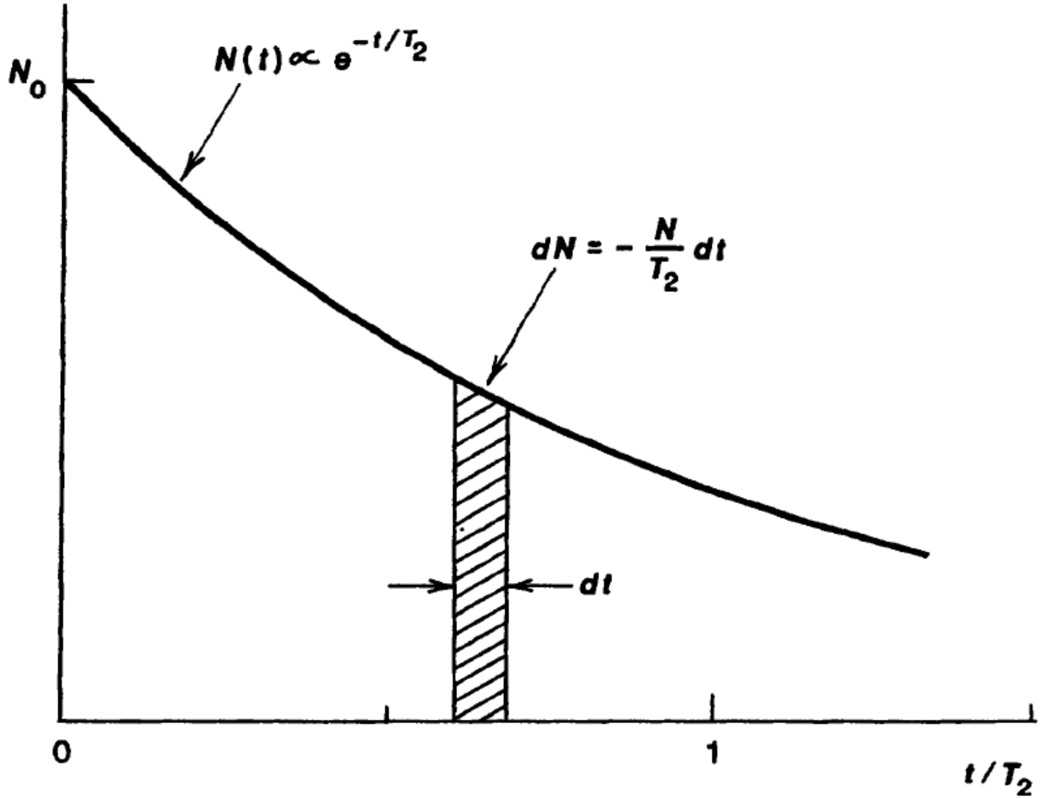
$$1/T_2$$

$$dN(t) = -\frac{N(t)}{T_2}\, dt.$$

$$N(t) = N_0 e^{-(t-t_0)/T_2}, \qquad t > t_0.$$

Siegman book p. 94-95

Decay of number of uncollided dipoles



$$p_x(t) = N(t)\mu_x(t)$$

$$= N_0 e^{-(t-t_0)/T_2} \times \mu_{x0} \exp\left[-(\gamma/2)(t-t_0) + j\omega_a(t-t_0) + j\phi_0\right]$$

$$= p_{x0} \exp\left[-(\gamma/2 + 1/T_2)(t-t_0) + j\omega_a(t-t_0) + j\phi_0\right].$$

Siegman book p. 95

Slightly tricky part

$$p_x(t) = N(t)\mu_x(t)$$

$$= N_0 e^{-(t-t_0)/T_2} \times \mu_{x0} \exp\left[-(\gamma/2)(t-t_0) + j\omega_a(t-t_0) + j\phi_0\right]$$

$$= p_{x0} \exp\left[-(\gamma/2 + 1/T_2)(t-t_0) + j\omega_a(t-t_0) + j\phi_0\right].$$

$$\left(\frac{\gamma}{2}\right) \quad \begin{pmatrix}\text{single-dipole}\\ \text{decay rate}\end{pmatrix} \quad \Rightarrow \quad \left(\frac{\gamma}{2} + \frac{1}{T_2}\right) \quad \begin{pmatrix}\text{macroscopic}\\ \text{polarization}\\ \text{decay rate}\end{pmatrix}.$$

$$\frac{d^2\mu_x(t)}{dt^2} + \gamma\frac{d\mu_x(t)}{dt} + \omega_a^2\mu_x(t) = (e^2/m)\mathcal{E}_x(t)$$

$$\Downarrow$$

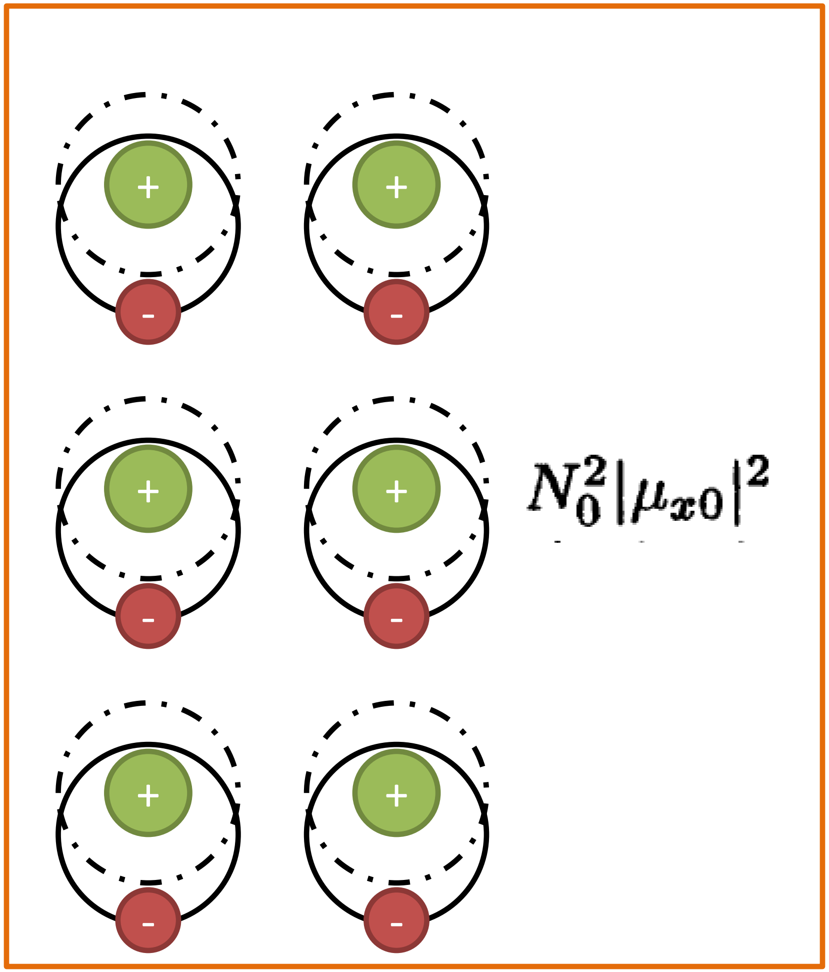
$$\frac{d^2p_x(t)}{dt^2} + (\gamma + 2/T_2)\frac{dp_x(t)}{dt} + \omega_a^2 p_x(t) = (Ne^2/m)\mathcal{E}_x(t),$$

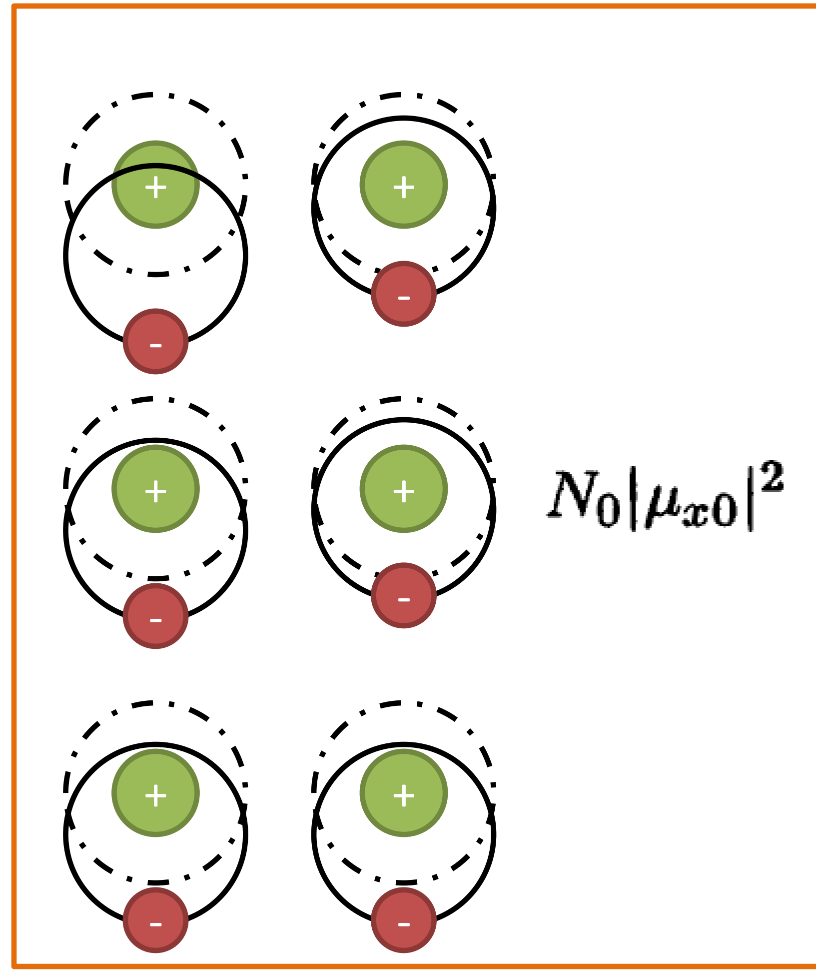
Siegman book p. 95-96

Mental exercise

$N_0 \mu_{x0},$ t = 0

t = T_2





Siegman book p. 100

The Lorentzian Line Shape

Normalized frequency shift:

$$\Delta x \equiv 2\frac{\omega - \omega_a}{\Delta\omega_a},$$

$$\tilde{\chi}_{\mathrm{at}}(\omega) = -j\chi_0''\frac{1}{1 + 2j(\omega - \omega_a)/\Delta\omega_a} = -j\chi_0''\frac{1}{1 + j\Delta x},$$

$$\chi_0'' \equiv \frac{Ne^2}{m\omega_a\epsilon\Delta\omega_a}$$

$$\tilde{\chi}_{\mathrm{at}}(\omega) \equiv \chi'(\omega) + j\chi''(\omega) = -\chi_0''\left[\frac{\Delta x}{1 + \Delta x^2} + j\frac{1}{1 + \Delta x^2}\right],$$

Siegman book p. 106

A plot

$$\chi_{res}(\omega) = \chi'(\omega) + i\,\chi''(\omega) = -\chi_0''\left[\frac{\Delta x}{1+(\Delta x)^2} + i\,\frac{1}{1+(\Delta x)^2}\right]$$

$$\chi''(\omega) = -\chi_0''\,\frac{1}{1+(\Delta x)^2}$$

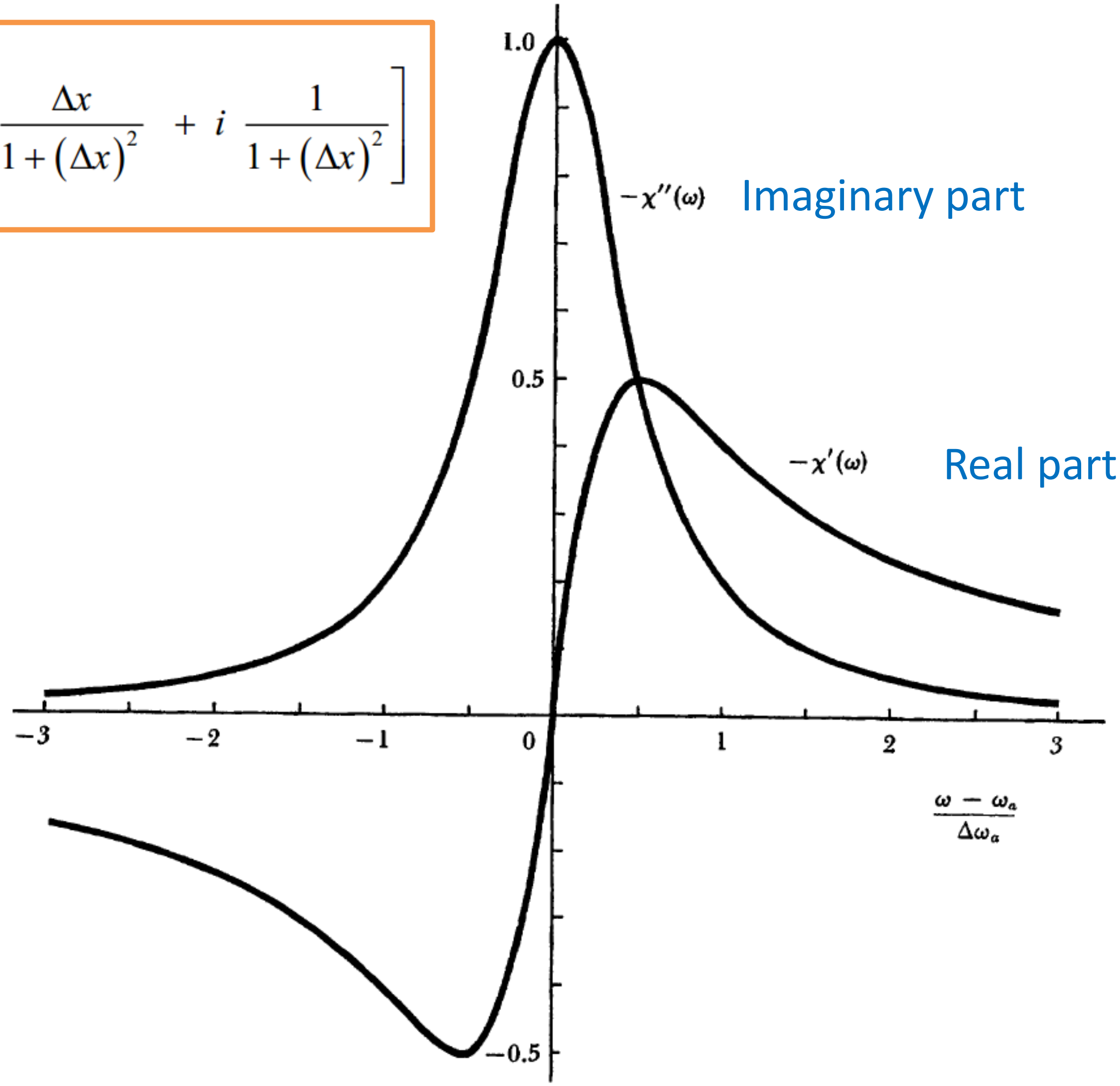
= absorbing part of the line shape

$$\chi'(\omega) = -\chi_0''\,\frac{\Delta x}{1+(\Delta x)^2}$$

= dispersive, phase shift part of the line shape

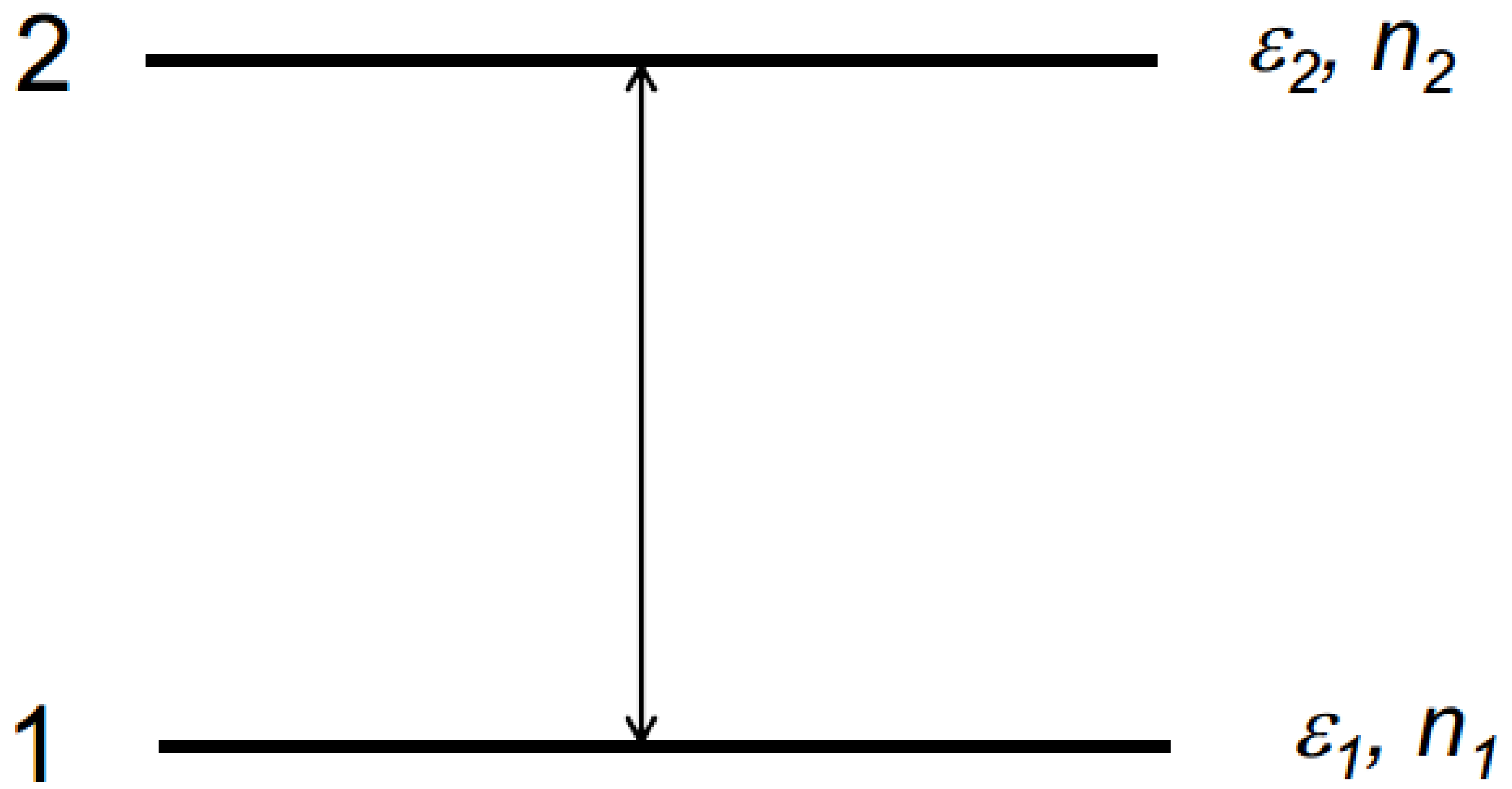
FWHM

$$\Delta\omega_a = \gamma + 2/T_2.$$



Siegman book p. 108

Finally atomic levels



Random picture from somewhere

Lets convert it to real atomic transitions

$$\chi_0'' \equiv \frac{Ne^2}{m\omega_a \epsilon \Delta\omega_a},$$

+

$$\gamma_{\mathrm{rad,ceo}} = \frac{e^2 \omega_a^2}{6\pi\epsilon m c^3}.$$

Ref– page 13

$$\chi_0'' = \frac{3}{4\pi^2} \frac{N\lambda^3 \gamma_{\mathrm{rad,ceo}}}{\Delta\omega_a}.$$

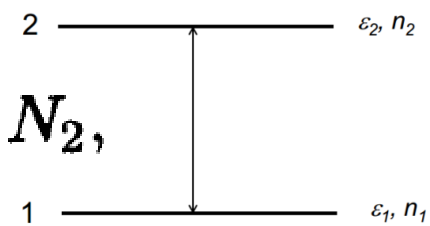
$$\omega = 2\pi f = 2\pi \frac{c}{\lambda}$$

Siegman book p. 110

Population difference

$$\chi_0'' = \frac{3}{4\pi^2}\frac{N\lambda^3\gamma_{\mathrm{rad,ceo}}}{\Delta\omega_a}.$$

$$\Delta N_{12} \equiv N_1 - N_2,$$



$$\tilde{\chi}_{\mathrm{at}}(\omega) = -j\frac{3}{4\pi^2}\frac{\Delta N\lambda^3\gamma_{\mathrm{rad}}}{\Delta\omega_a}\frac{1}{1+2j(\omega-\omega_a)/\Delta\omega_a}.$$

$$\begin{aligned}\tilde{\chi}_{\mathrm{at}}(\omega) &= -j\chi_0'' \times \frac{1}{1+2j(\omega-\omega_a)/\Delta\omega_a} \\ &= -\chi_0''\left[\frac{2(\omega-\omega_a)/\Delta\omega_a}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2} + j\frac{1}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2}\right],\end{aligned}$$

Siegman book p. 110-111

Quantum susceptibility equation

$$\chi_0'' = \frac{3}{4\pi^2}\frac{\Delta N \lambda^3 \gamma_{\text{rad}}}{\Delta\omega_a}.$$

Replace CEO with actual radiative decay rate

Classical $N\lambda^3$ Quantum $\Delta N \lambda^3$

$$\lambda \equiv \lambda_0/n$$

γ_{rad}

Spontaneous emission

inverse linewidth $1/\Delta\omega_a$

is proportion to

$$\Delta N \lambda^3 \gamma_{\text{rad}}$$

$$\tilde{\chi}_{\text{at}}(\omega) \equiv \chi'(\omega) + j\chi''(\omega)$$

Frequency variations

Quantum polarization equation

CEO

$$\frac{d^2 p_x(t)}{dt^2} + (\gamma + 2/T_2)\frac{dp_x(t)}{dt} + \omega_a^2 p_x(t) = (Ne^2/m)\mathcal{E}_x(t),$$

Quantum

$$\frac{d^2 p_x(t)}{dt^2} + \Delta\omega_a \frac{dp_x(t)}{dt} + \omega_a^2 p_x(t) = \frac{3\omega_a \epsilon \lambda^3 \gamma_{\mathrm{rad}}}{4\pi^2} \Delta N(t)\mathcal{E}_x(t),$$

Siegman book p. 112

Substitutions #1

1 Transition frequency

$$\omega_a \Rightarrow \omega_{ji} = \frac{E_j - E_i}{\hbar}.$$

2 Atomic population difference

$$\Delta N \Rightarrow \Delta N_{ij} = N_i - N_j,$$

3 Radiative decay time

$$\gamma_{\mathrm{rad}} \Rightarrow \gamma_{\mathrm{rad},ji}.$$

$$\gamma_{\mathrm{rad},ji} \equiv A_{ji}$$

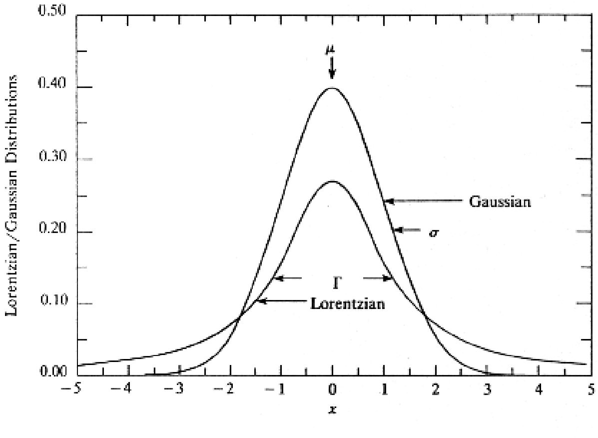
4 Transition linewidth

$$\Delta\omega_a \Rightarrow \Delta\omega_{a,ij}.$$

Siegman book p. 113-114

Substitutions #2

5 Transition lineshape



6 Tensor properties

$$\boldsymbol{E}(\omega) = [\tilde{E}_x, \tilde{E}_y, \tilde{E}_z].$$

7 Polarization properties

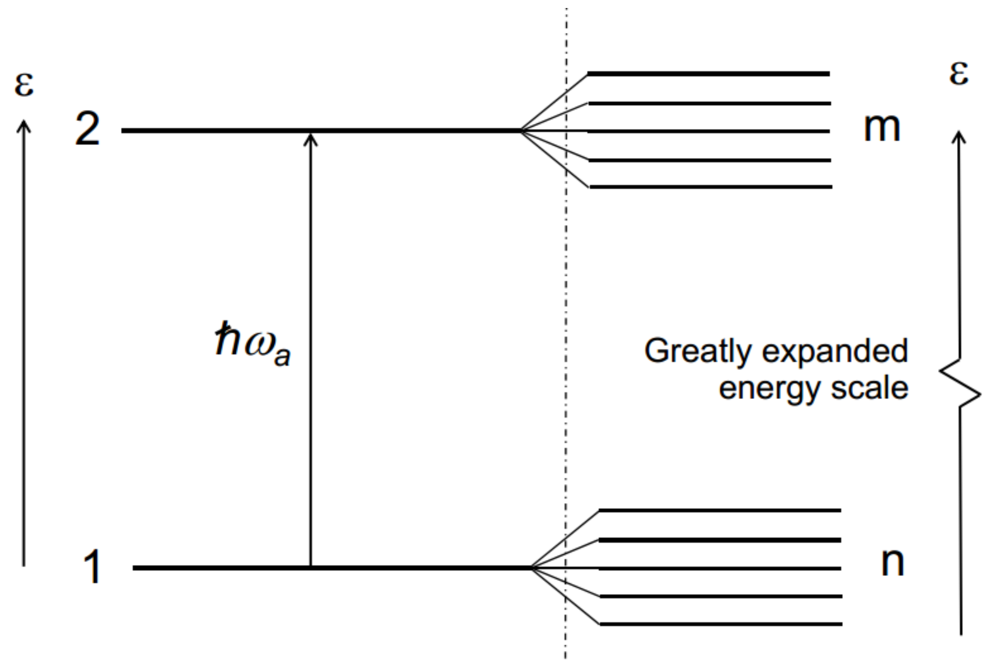
$$\frac{3}{4\pi^2} \Rightarrow \frac{3^*}{4\pi^2},$$

Siegman book p. 113-114

Substitutions #3

8 Degeneracy effects

$$\Delta N_{ij} = (N_i - N_j) \Rightarrow \Delta N_{ij} = (g_j/g_i)\, N_i - N_j,$$



9 Inhomogeneous broadening

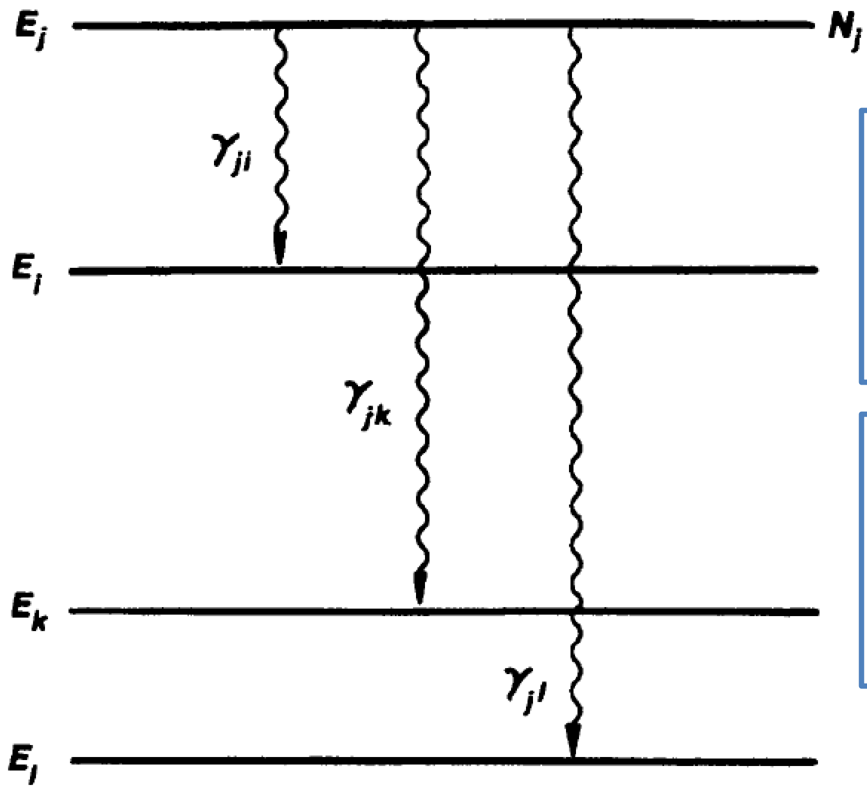
$$\Delta\omega_a = \gamma + 2/T_2.$$

$$\Delta\omega_a \Rightarrow \Delta\omega_d.$$

Siegman book p. 113-114

ELECTRIC-DIPOLE TRANSITIONS IN REAL ATOMS

Energy decay in real atoms



"Rate equation"

$$\frac{dN_j}{dt} = -\sum_{E_i<E_j} \gamma_{ji} N_j = -\gamma_j N_j = -N_j/\tau_j,$$

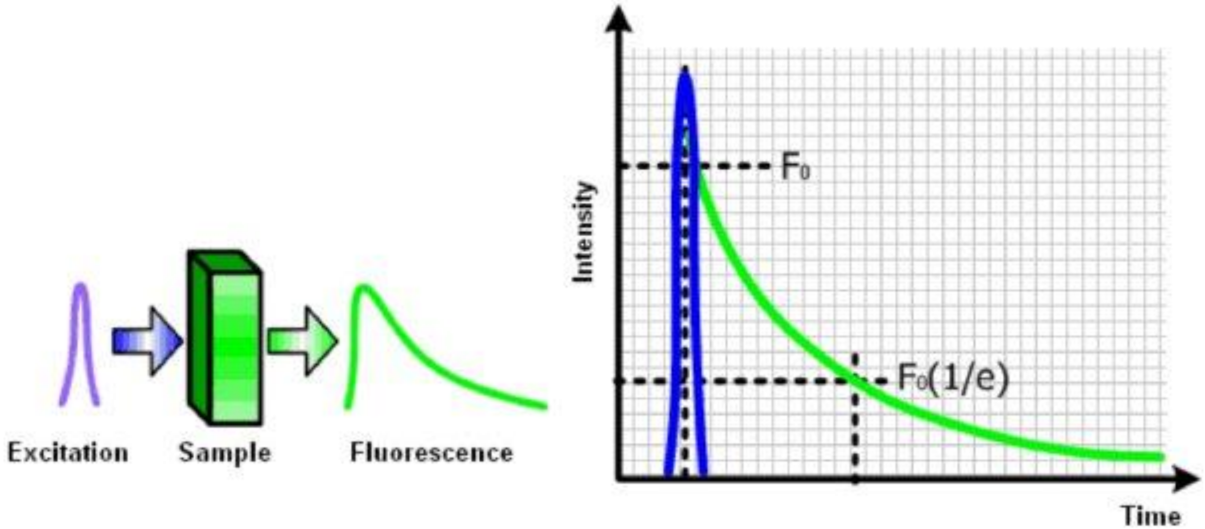
"Decay rate"

$$\gamma_j \equiv \frac{1}{\tau_j} = \sum_{E_i<E_j} \gamma_{ji} = \sum_{E_i<E_j} \left[\gamma_{\mathrm{rad},ji} + \gamma_{\mathrm{nr},ji}\right],$$

$$N_j(t) = N_j(t_0)e^{-\gamma_j(t-t_0)} = N_j(t_0)e^{-(t-t_0)/\tau_j}.$$

Siegman book p. 118-119

Fluorescent Lifetime Measurements

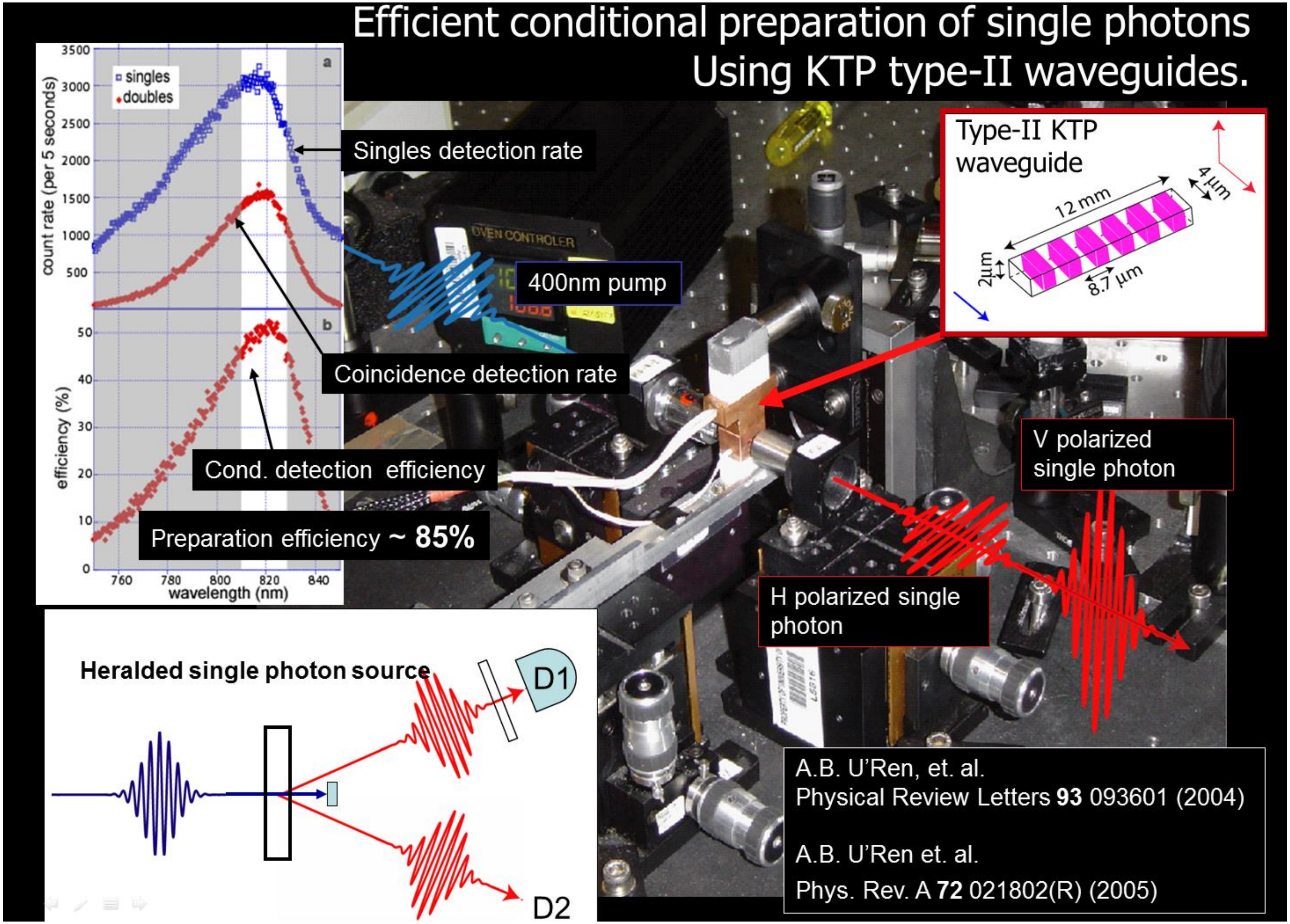


$$I_{fl}(t) = \text{const} \times \gamma_{\mathbf{rad},ji} N_j(t).$$

$$I_{fl}(t) = \text{const} \times N_j(t) = \text{const} \times e^{-t/\tau_j}.$$

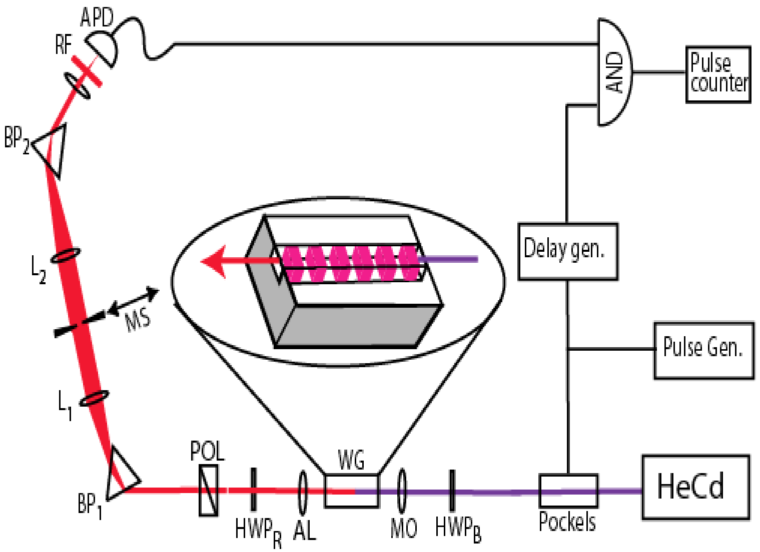
Siegman book p. 119-120

Example #1



Without authors permission

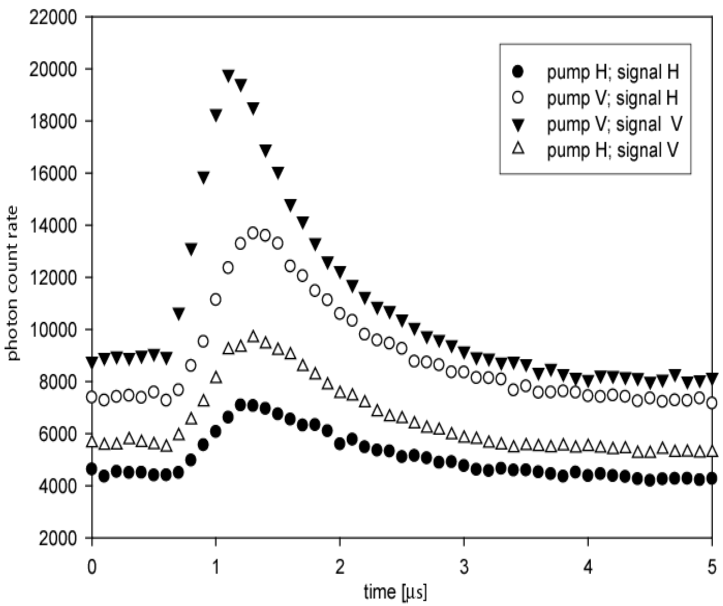
Temporally-resolved fluorescence measurements



•Pulsed pump
[HeCd laser + pockels cell]

•Time-gated photon counting

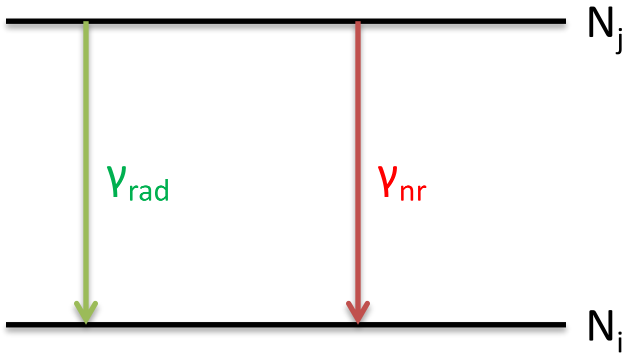
Can resolve fluorescence lifetime



Fluorescence lifetime ~ 1-10 µs
[depending on waveguide]

Some unpublished data

Radiative and non radiative decay. Again…



Total decay rate

$$\gamma_{ji} = \gamma_{\text{rad},ji} + \gamma_{\text{nr},ji}.$$

Einstein A coefficients

$$\gamma_{\text{rad},ji} \equiv A_{ji},$$

$$A_{ji} = \frac{8\pi^2}{\epsilon \hbar \lambda^3} \left| \iiint \psi_j^*(\boldsymbol{r})\, e\boldsymbol{r}\, \psi_i(\boldsymbol{r})\, d\boldsymbol{r} \right|^2,$$

$$\gamma_{\text{rad,ceo}} = \frac{e^2 \omega_a^2}{6\pi\epsilon m c^3} \approx 2.47 \times 10^{-22} \times n f_a^2,$$

$$\tau_{\text{rad,ceo}}(\text{ns}) \approx \frac{45 \times [\lambda_0(\text{microns})]^2}{n},$$

Siegman book p. 120-121

Oscillator strength

$$\mathcal{F}_{ji} \equiv \frac{\gamma_{\text{rad},ji}}{3\gamma_{\text{rad,ceo}}} = \frac{\tau_{\text{rad,ceo}}}{3\tau_{\text{rad},ji}}.$$

"General" rule

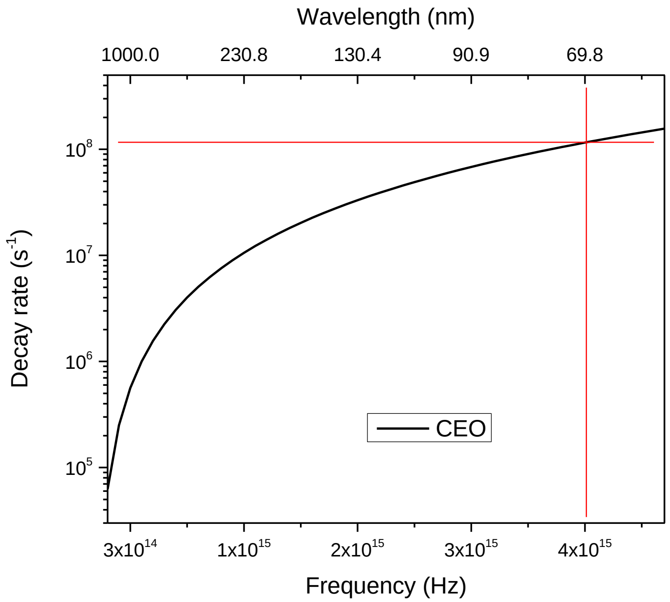
$$\gamma_{\text{rad},ji} \leq \gamma_{\text{rad,ceo}},$$

Examples:

Transition	*Wavelength*	*Radiative decay rate*	*Oscillator strength*	*Comments*
Neodymium YAG laser transition:				
${}^4F_{3/2} \rightarrow {}^4I_{3/2}$	1.064 μm	820 s^{-1} (1.22 ms)	$\approx 8 \times 10^{-6}$	Measured τ_2 is 230 μs
Ruby laser transition:				
${}^2E \rightarrow {}^4A_2$	694 nm	230 s^{-1} (4.3 ms)	$\approx 10^{-6}$	Decay is almost purely radiative

Siegman book p. 121-123

Radiative decay rates



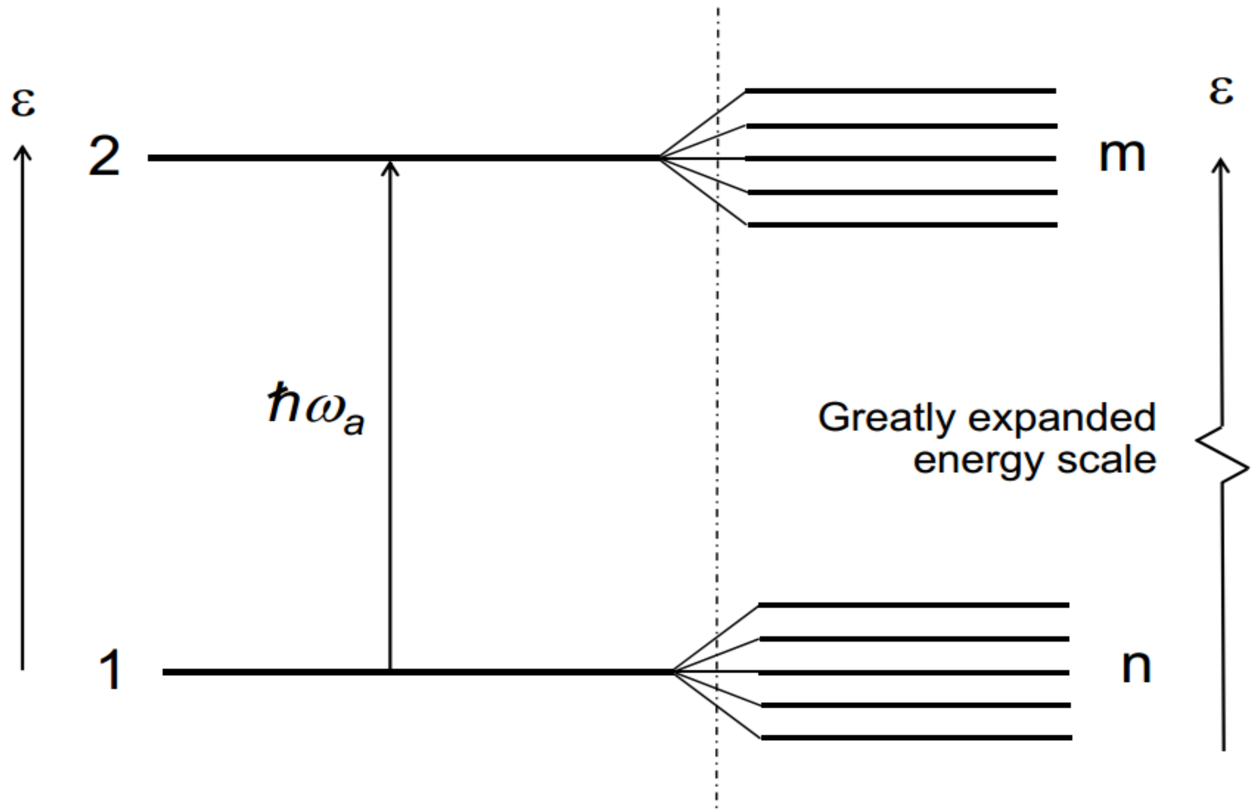
$$\gamma_{\text{rad,ceo}} = \frac{e^2\omega_a^2}{6\pi\epsilon m c^3}.$$

Parameter	Value
e	1.602e-19 (C)
ε	8.85e-12 (F/m)
m	9.11e-31 (kg)
c	3e8 (m/s)

This classical oscillator radiative decay rate has a value $\gamma_{\text{rad,ceo}} \approx 10^8$ sec^{-1} for a visible frequency oscillator, compared to an oscillation frequency of $\omega_a \approx 4 \times 10^{15}$ sec^{-1}.

Siegman book p. 84

Level degeneracy #1



$$\mathcal{F}_{ji}\big|_{\text{down}} = -\frac{\gamma_{\text{rad},ji}}{3\gamma_{\text{rad,ceo}}} \qquad \mathcal{F}_{ij}\big|_{\text{up}} = +\frac{g_j}{g_i}\frac{\gamma_{\text{rad},ji}}{3\gamma_{\text{rad,ceo}}}.$$

Siegman book p. 123

Level degeneracy #2

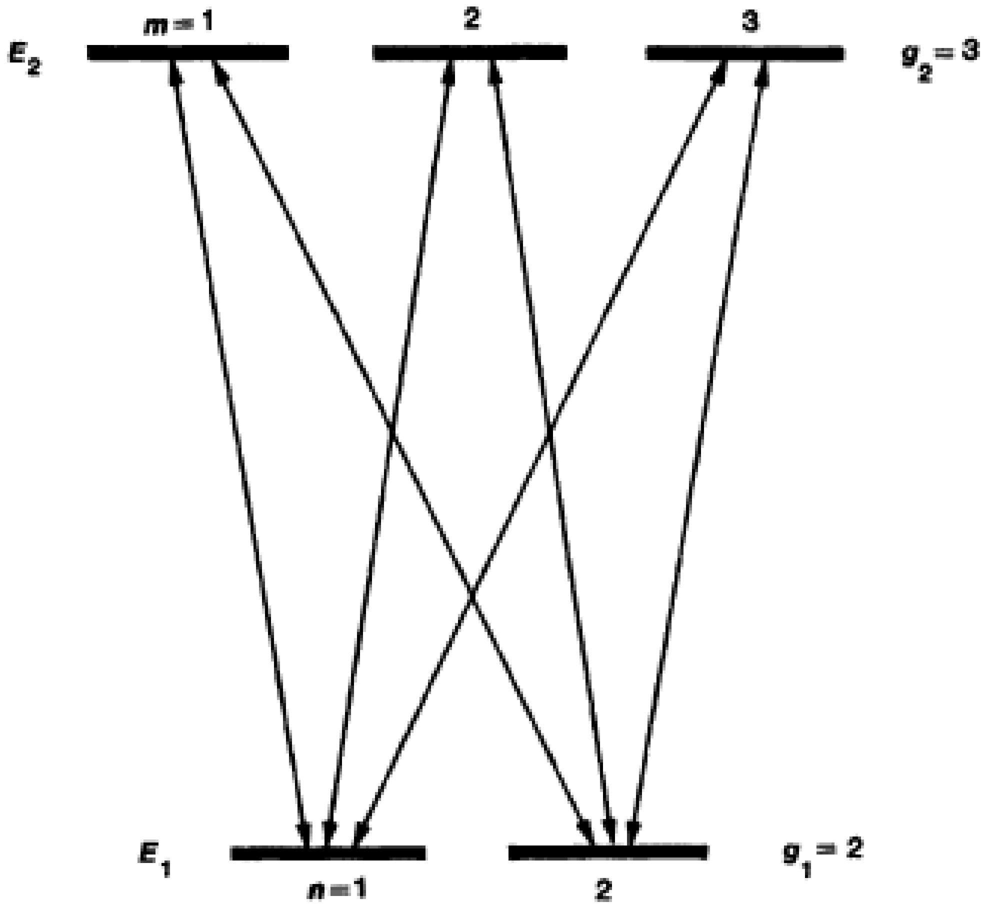
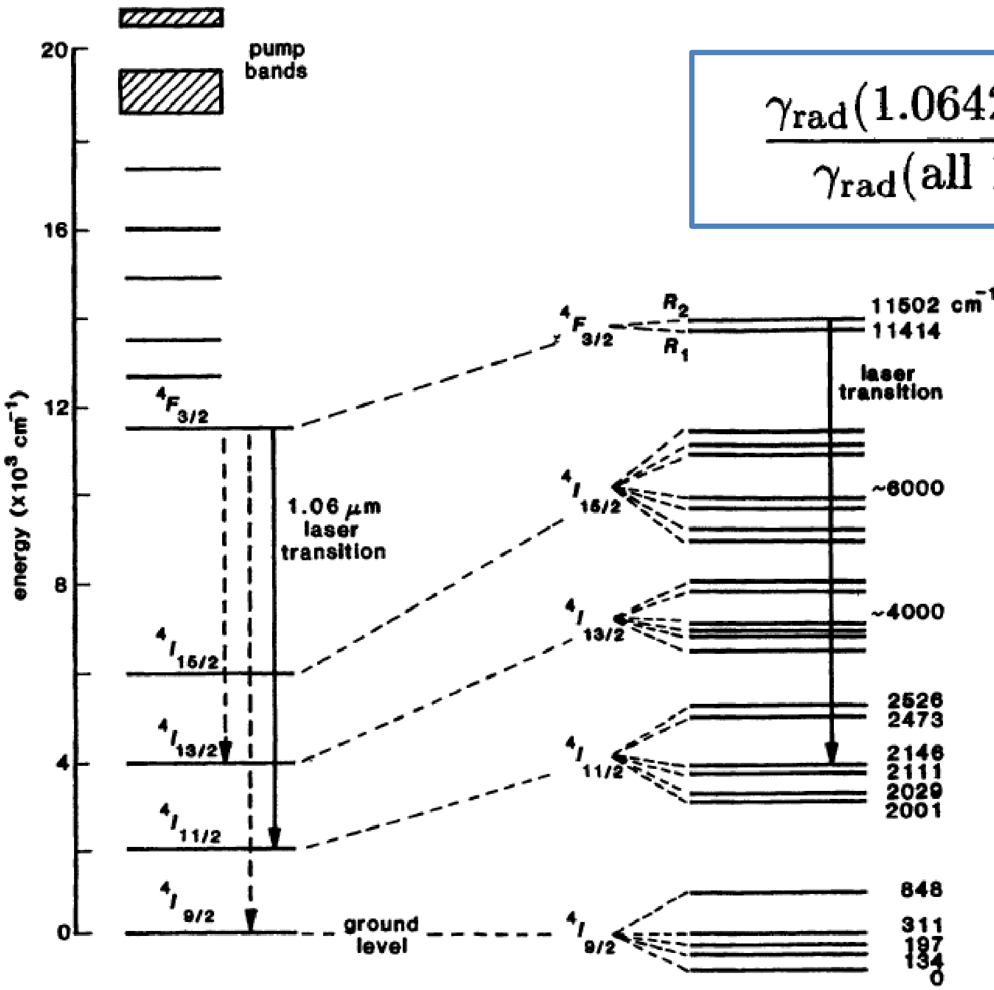


FIGURE 3.12
Degenerate sublevels of two quantum energy levels E_1 and E_2. Each sublevel is a separate and distinct quantum energy eigenstate, but the degenerate sublevels all have the same energy eigenvalue.

Siegman book p. 155

Nd:YAG transitions



$$\frac{\gamma_{\text{rad}}(1.0642\ \mu\text{m laser line}) \times N_{2b}}{\gamma_{\text{rad}}(\text{all } 1.06\ \mu\text{m lines}) \times N_2} \approx 0.135.$$

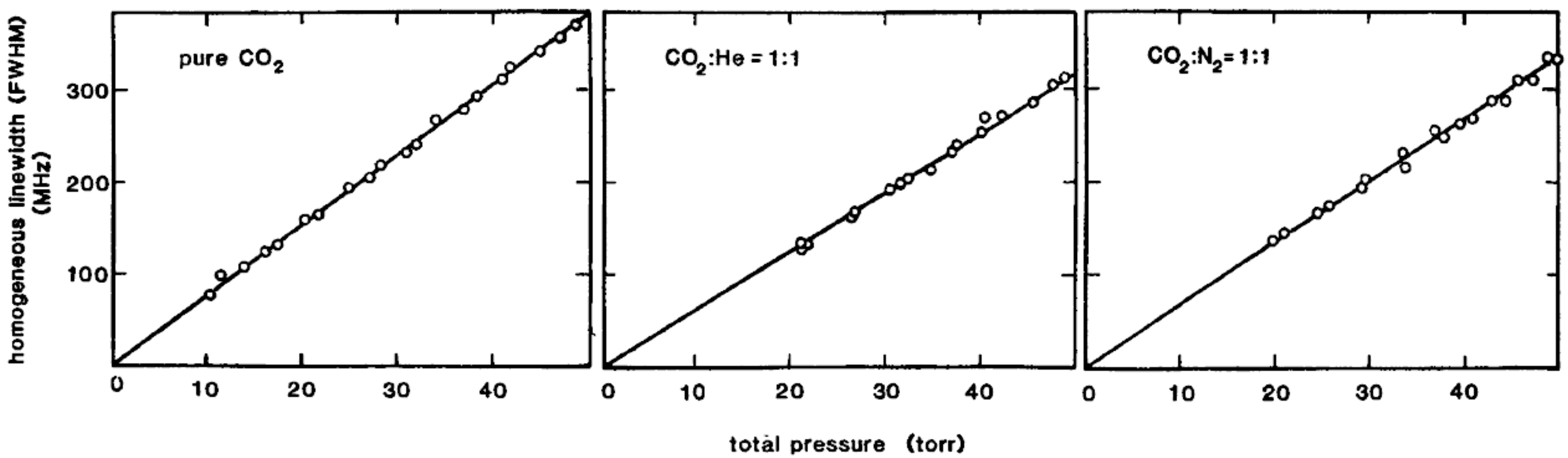
$$\gamma_{\text{rad}}(1.0642\ \mu\text{m}) \approx (0.135/0.40) \times 2.435 \times 10^3 \approx 820\ \text{sec}^{-1}.$$

Siegman book p. 123-125

Line-broadening mechanisms in real atoms

$$\Delta\omega_a = \gamma + 2/T_2,$$ → $$\Delta\omega_a = \gamma_i + \gamma_j + 2/T_{2,ij},$$

CEO | Real atoms



Pressure broadening in gases

Homogeneous linewidth dependence: $$\Delta\omega_a = A + BP,$$

Siegman book p. 126-129

Pressure-broadening coefficients

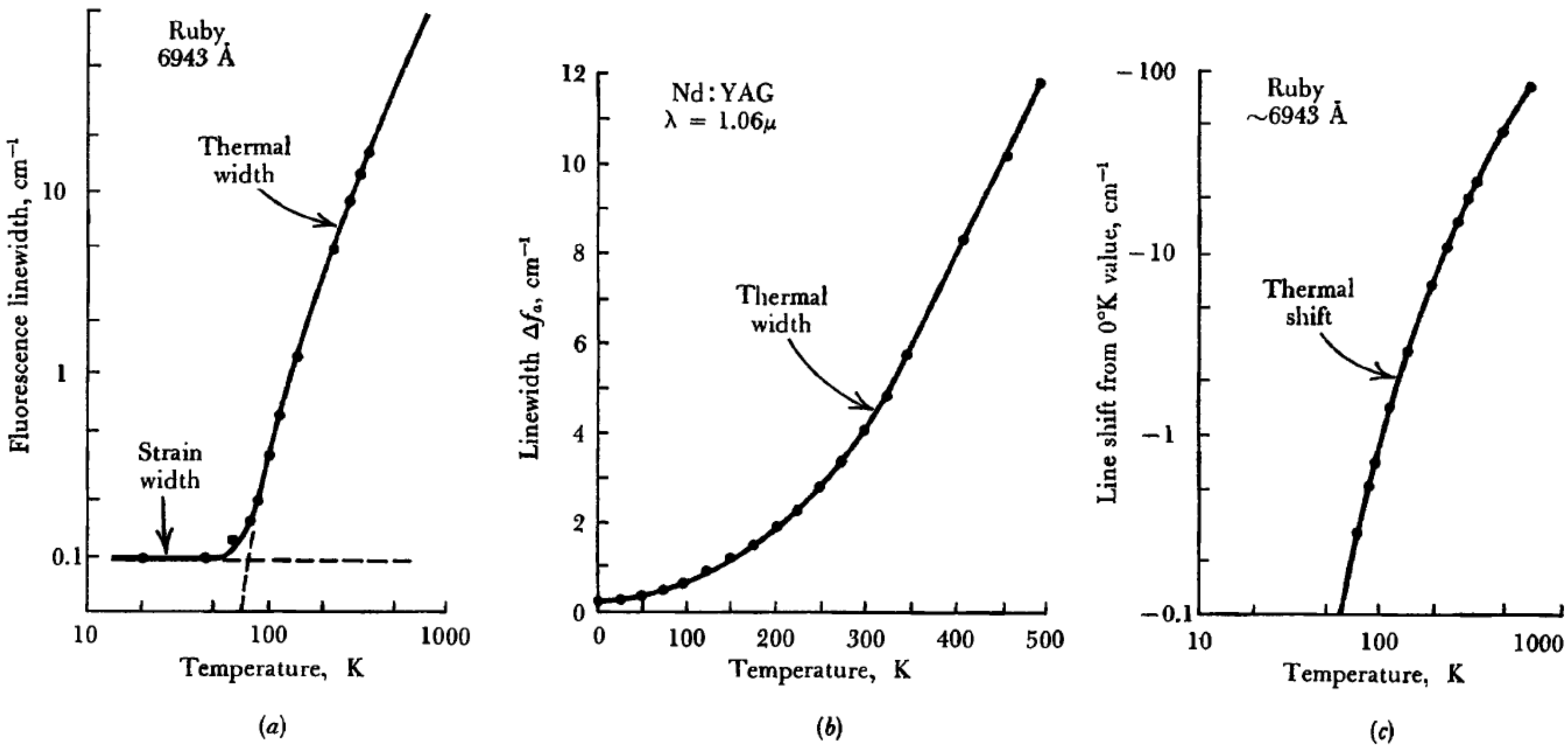
Wavelength	Collision partners	Pressure broadening
Mercury resonance line:		
2537Å	Hg + Ar,N_2,CO_2	10–20 MHz/torr
Sodium resonance line:		
589 nm	Na + Na	≈ 2000 MHz/torr
He-Ne laser transitions:		
633 nm	He+Ne	≈ 70 MHz/torr
3.39 μm	He+Ne	50–80 MHz/torr
CO_2 laser transition:		
10.6 μm	CO_2 + CO_2	7.6 MHz/torr (5.8 GHz/atm)
10.6 μm	CO_2 + N_2	5.5 MHz/torr (4.2 GHz/atm)
10.6 μm	CO_2 + He	4.5 MHz/torr) (3.5 GHz/atm)
10.6 μm	CO_2 + H_2O	2.9 MHz/torr (2.2 GHz/atm)

Relationship between partial pressure and density of each species in gas mixture.

$$N(\text{atoms/cm}^3) = 9.65 \times 10^{18} \frac{P(\text{torr})}{T(K)}$$

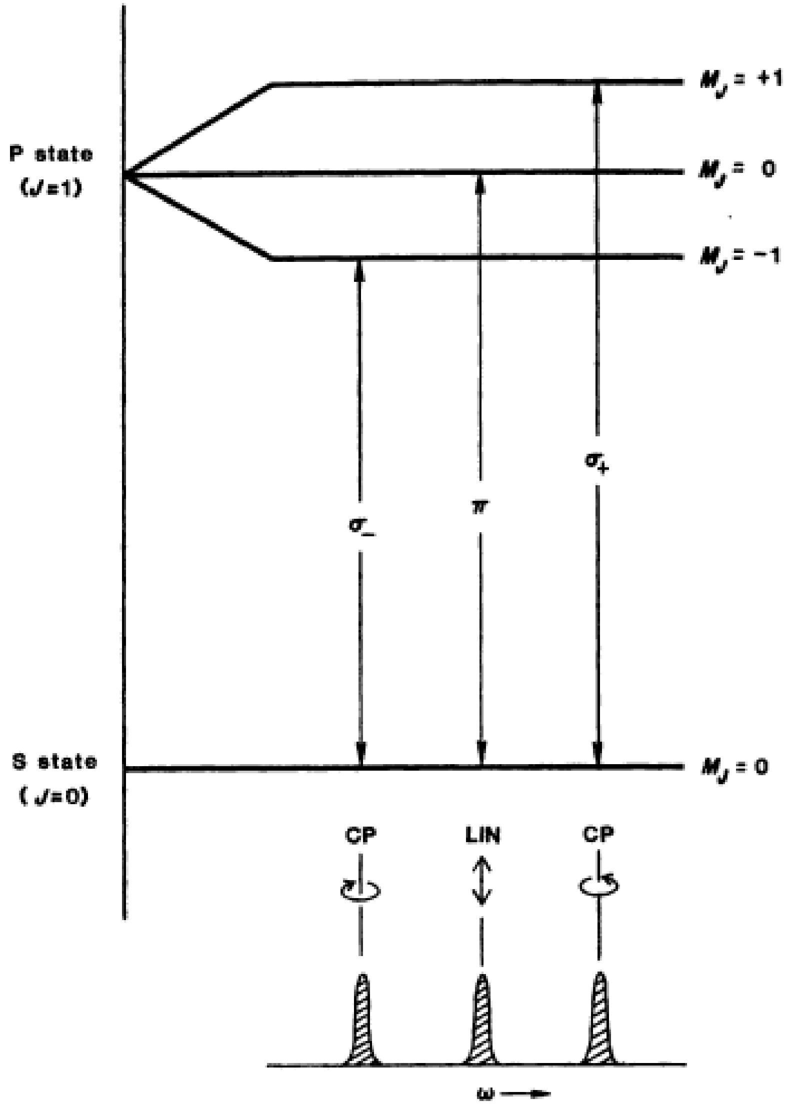
Siegman book p. 129

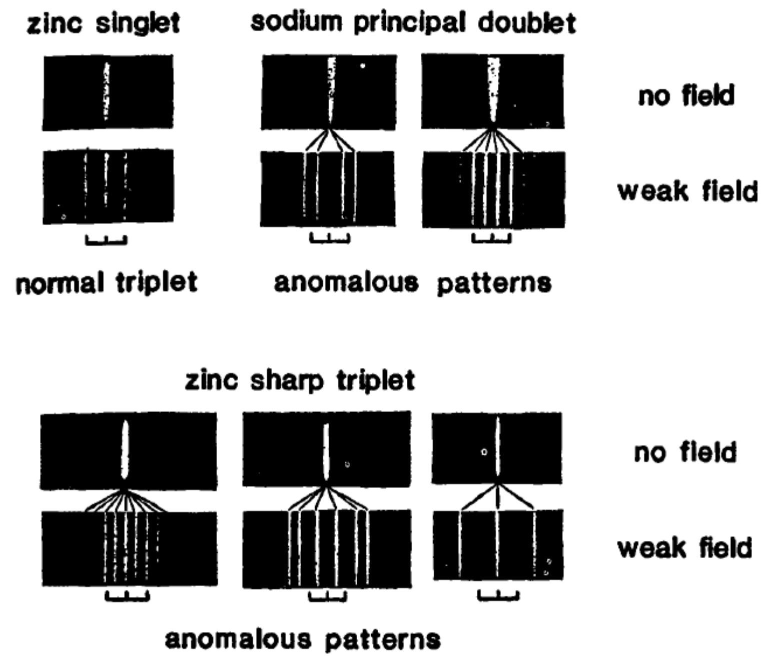
Phonon broadening



Siegman book p. 130-131

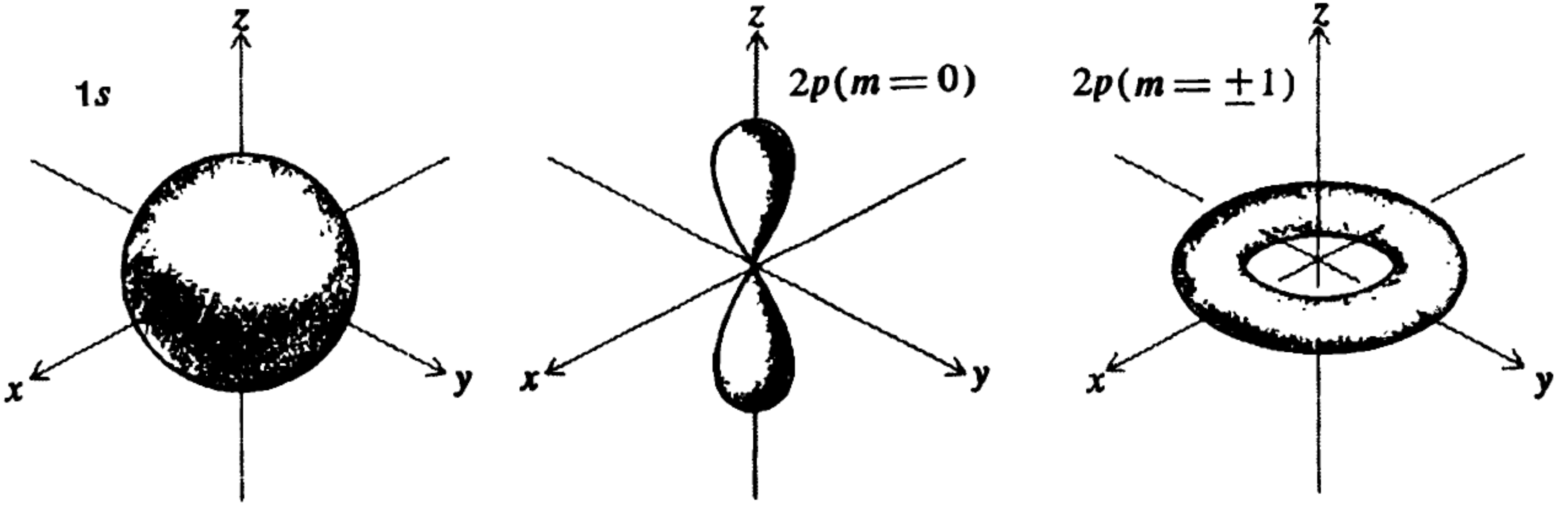
Zeeman-split atomic transitions





Siegman book p. 136-137

Electron density distribution in atoms



- The electron density is proportional to the product of the wavefunction and its complex conjugate.
- Since the electron density distribution does not change with time, the atom does not radiate in these stationary states.

Siegman book p. 139

Mathematical interpretation of the picture

$$\rho(\boldsymbol{r},t) = \left|\tilde{a}_1(t)e^{-iE_1t/\hbar}\psi_1(\boldsymbol{r}) + +\tilde{a}_2(t)e^{-iE_2t/\hbar}\psi_2(\boldsymbol{r})\right|^2$$

$$= |\tilde{a}_1(t)|^2\ |\psi_1(\boldsymbol{r})|^2 + |\tilde{a}_2(t)|^2\ |\psi_2(\boldsymbol{r})|^2$$

$$+ \tilde{a}_1(t)\tilde{a}_2^*(t)\psi_1(\boldsymbol{r})\psi_2^*(\boldsymbol{r})\exp[i(E_2 - E_1)t/\hbar]$$

$$+ \tilde{a}_1^*(t)\tilde{a}_2(t)\psi_1^*(\boldsymbol{r})\psi_2(\boldsymbol{r})\exp[-i(E_2 - E_1)t/\hbar]$$

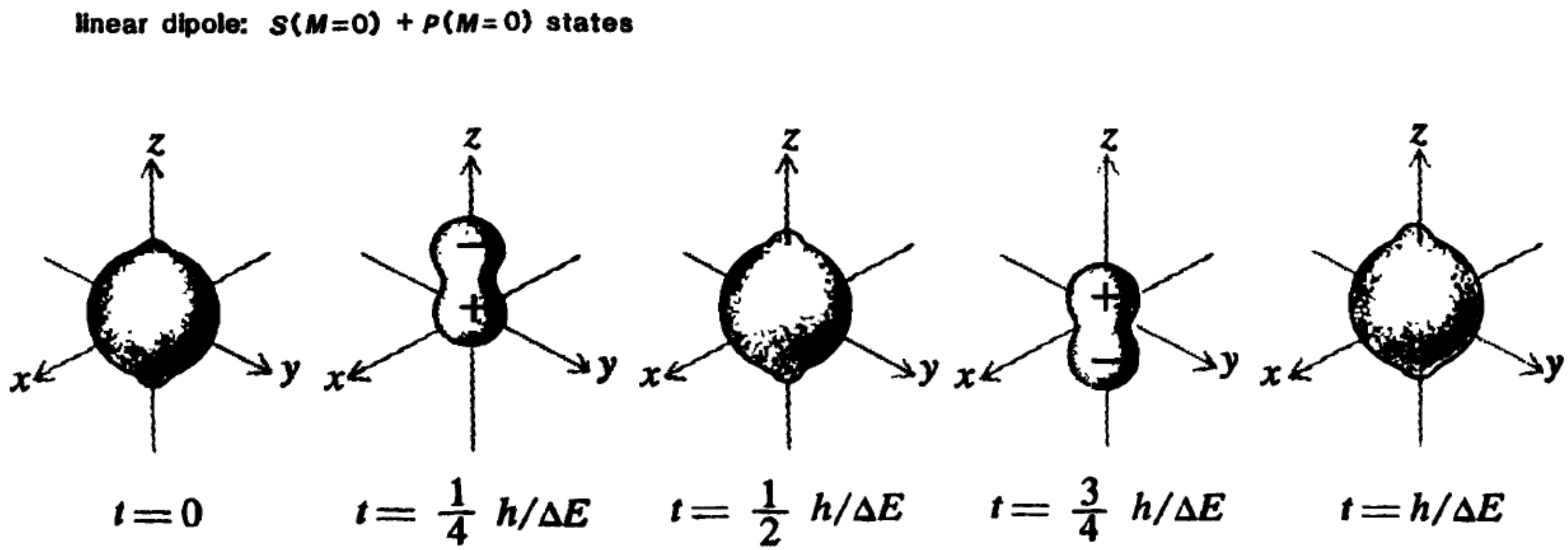
$$= \rho_{dc}(\boldsymbol{r}) + \rho_{ac}(\boldsymbol{r},t).$$

x2 **Errata**

$$\rho_{ac}(\boldsymbol{r},t) = \mathrm{Re}\left[\tilde{a}_1(t)\tilde{a}_2^*(t)\psi_1(\boldsymbol{r})\psi_2^*(\boldsymbol{r})e^{i\omega_{21}t}\right].$$

Siegman book p. 138

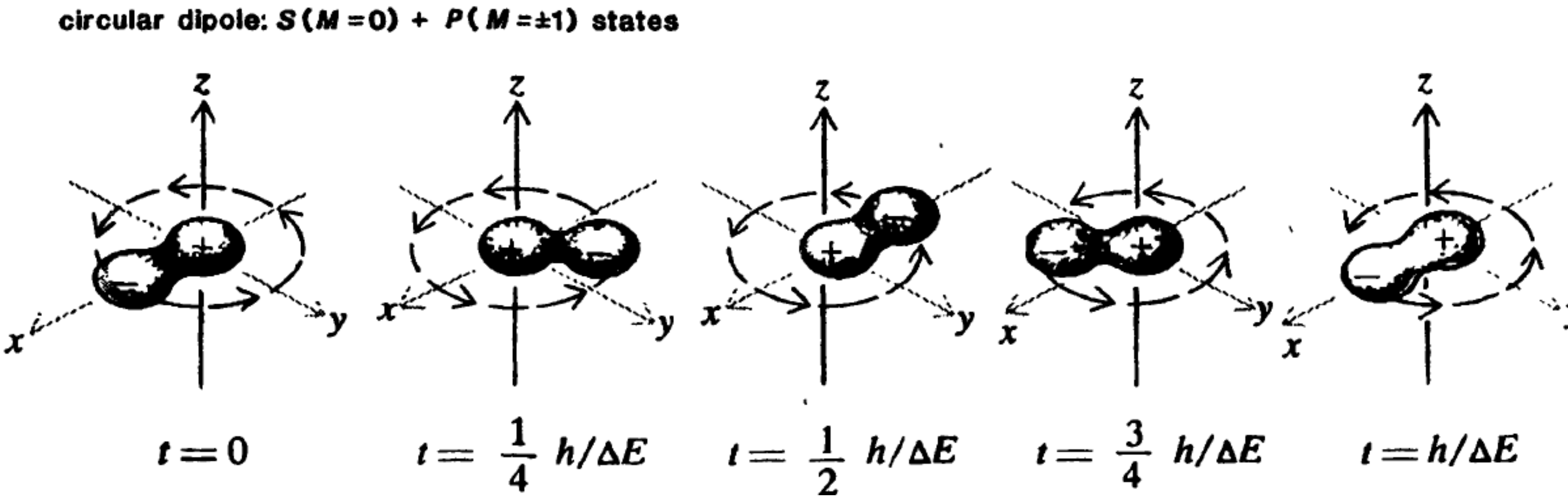
Linear dipole



Laser radiation that is linearly polarized in the z-direction couples very efficiently with superposition states with the same M_J value.

Siegman book p. 140

Circular dipole

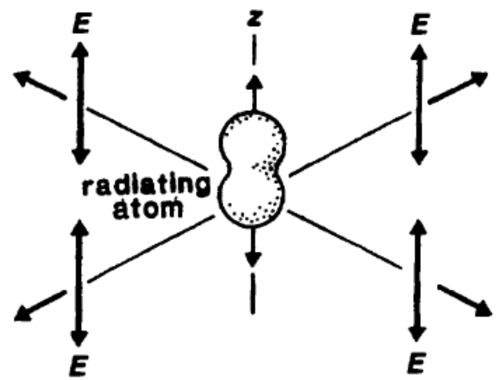


Laser radiation that is circularly polarized in the x-y plane couples very efficiently with superposition states with MJ value that differ by +1 or -1.

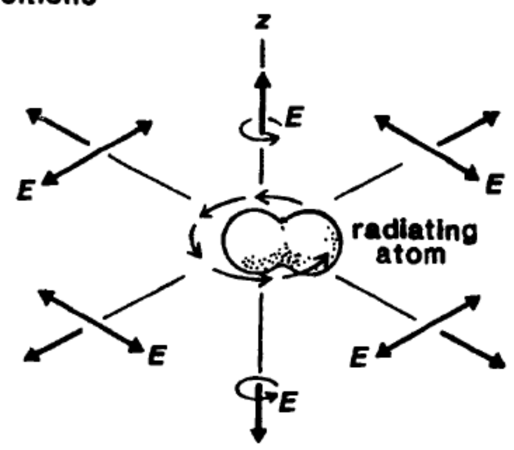
Siegman book p. 140

Linear and circular micro-summary

π transitions



σ transitions



Atom whose charge distribution can oscillate only in a certain direction on a given transition will obviously respond only to applied fields that have the same direction or sense of polarization

Tensor susceptibilities

$$\boldsymbol{P}(\omega) = \chi(\omega)\epsilon\boldsymbol{E}(\omega),$$

Linear

$$\boldsymbol{P}(\omega) = \begin{bmatrix} \tilde{P}_x(\omega) \\ \tilde{P}_y(\omega) \\ \tilde{P}_z(\omega) \end{bmatrix} = \tilde{\chi}(\omega)\epsilon \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_x(\omega) \\ \tilde{E}_y(\omega) \\ \tilde{E}_z(\omega) \end{bmatrix}$$

Circular

$$\boldsymbol{P} = \begin{bmatrix} \tilde{P}_x \\ \tilde{P}_y \\ \tilde{P}_z \end{bmatrix} = \tilde{\chi}(\omega)\epsilon \times \frac{3}{2} \begin{bmatrix} 1 & \mp j & 0 \\ \pm j & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \end{bmatrix}.$$

Siegman book p. 143-145

The "factor of three"

TABLE 3.3
Normalized tensor responses

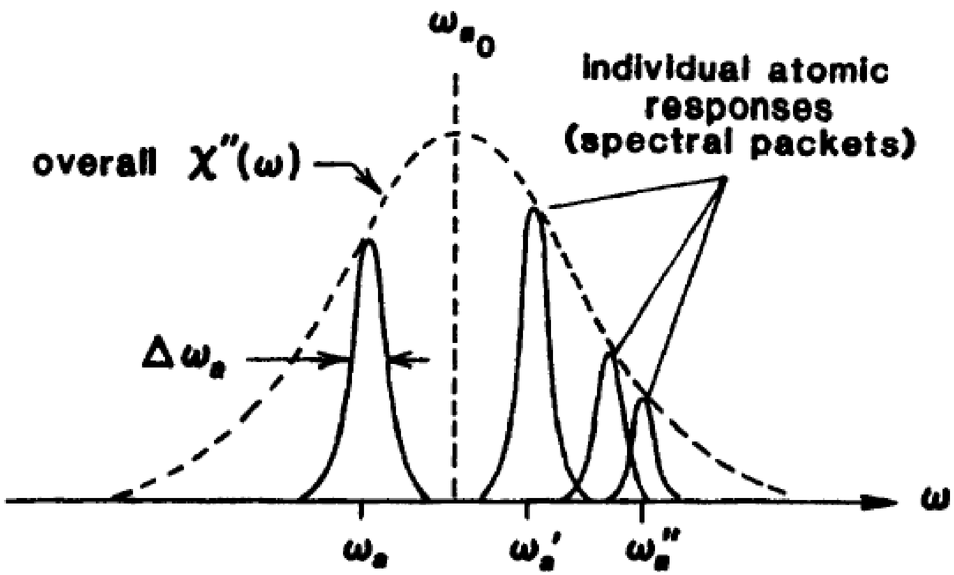
Saturated Tensor Form	*Gain Applied Field Polarization*	*Normalized Response*
Circular, $x \to \pm y$	Circular, $x \to \pm y$	3
Circular, $x \to \pm y$	Circular, $x \to \mp y$	0
Circular, $x \to \pm y$	Linear (x or y)	1.5
Circular, $x \to \pm y$	Linear (x)	0
Circular, $x \to \pm y$	Random	1
Linear (x)	Linear (x)	3
Linear (x)	Linear (angle θ from x)	$3\cos^2\theta$
Linear (x)	Circular, $x \to \pm y$	1.5
Linear	Random	1
Linear (x)	Linear (y or x)	0
Isotropic	Arbitrary	1

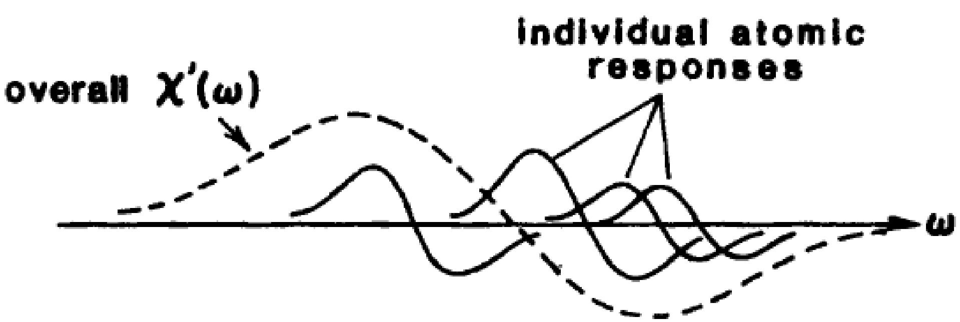
Siegman book p. 150-153

Inhomogeneous line broadening

Siegman book p. 158

Convolution of inhomogeneous and homogeneous profiles





Gasses

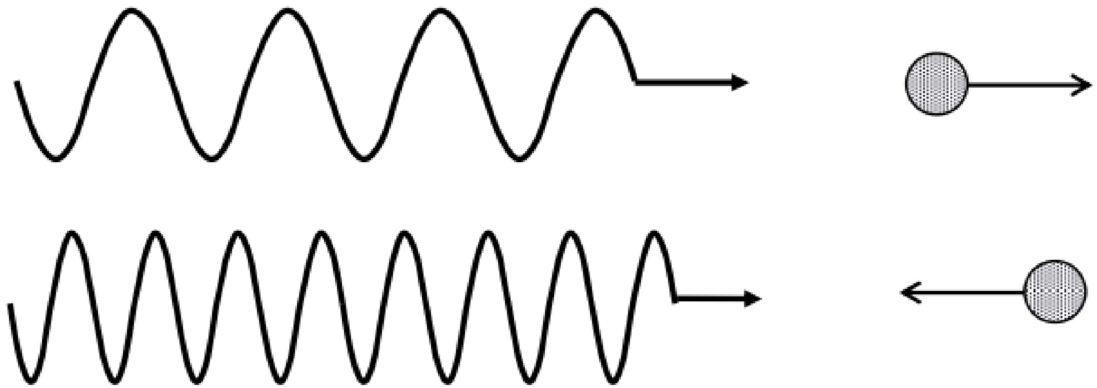
Different atoms will have different kinetic velocities through space

Solids

Atoms at different sites in a crystal may see slightly different local surroundings, or different local crystal structures, because of defects, dislocations, or lattice impurities

Siegman book p. 158-159

Example – Doppler broadening



$$\omega' = (1 - v_z/c)\,\omega. \qquad \omega_a = (1 + v_z/c)\,\omega_{a0}.$$

Average Doppler shift

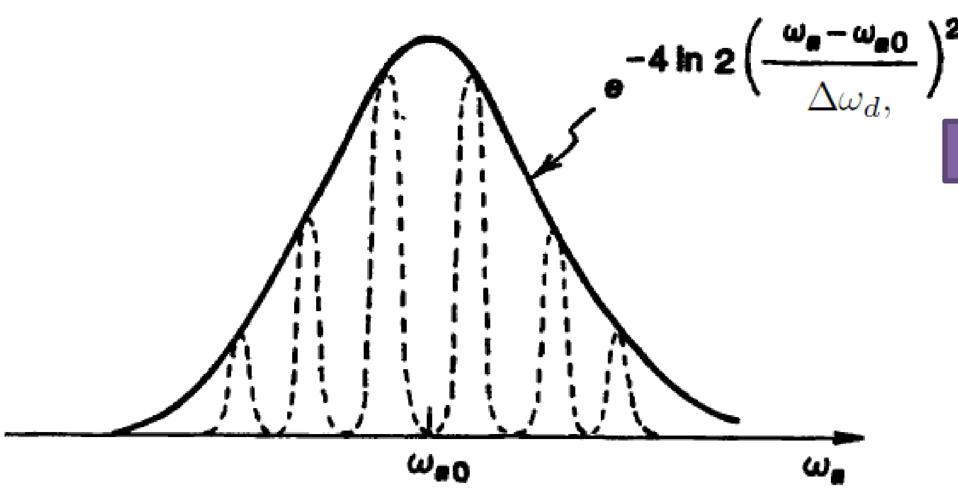
$$\frac{\omega_a - \omega_{a0}}{\omega_{a0}} \approx \sqrt{\frac{kT}{Mc^2}} \approx 10^{-6} \quad \left(\begin{array}{c}\text{for typical atomic}\\ \text{masses and temperatures}\end{array}\right).$$

Siegman book p. 160

Doppler lineshape

Gaussian distribution in gasses

$$g(v_z) = \left(\frac{1}{2\pi\sigma_v^2}\right)^{1/2} \exp\left(-\frac{v_z^2}{2\sigma_v^2}\right)$$

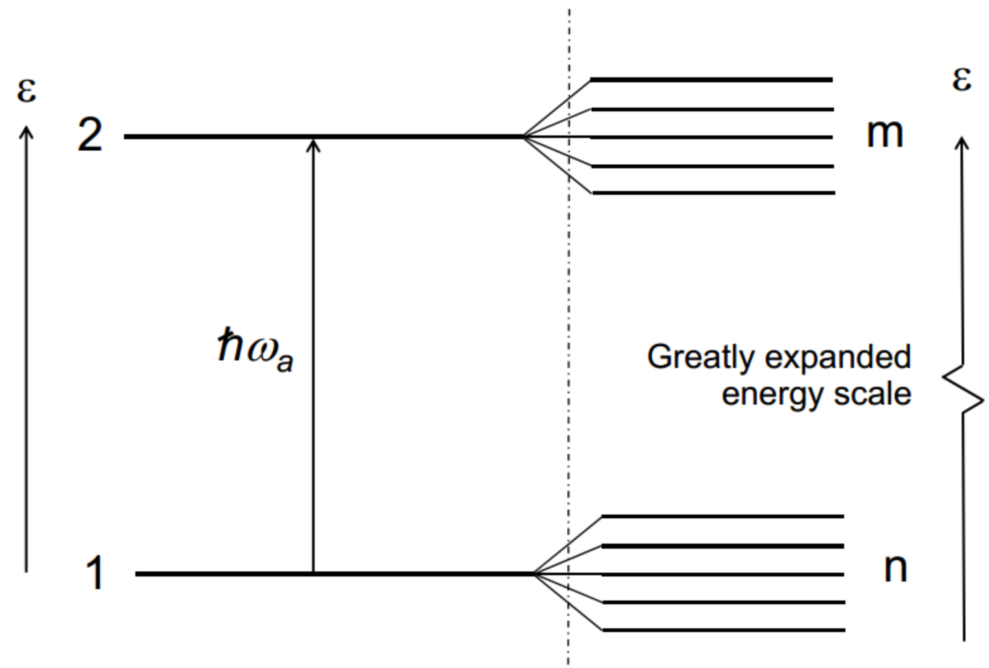


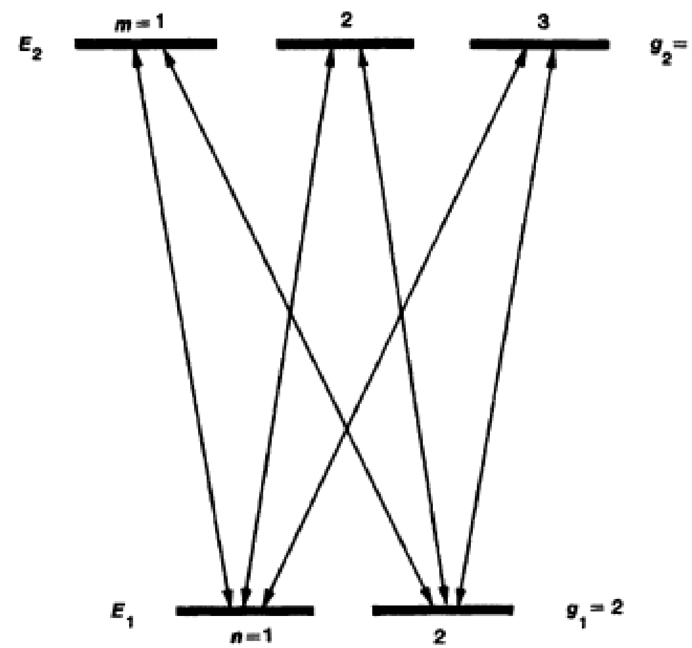
Errata

Siegman book p. 160-161

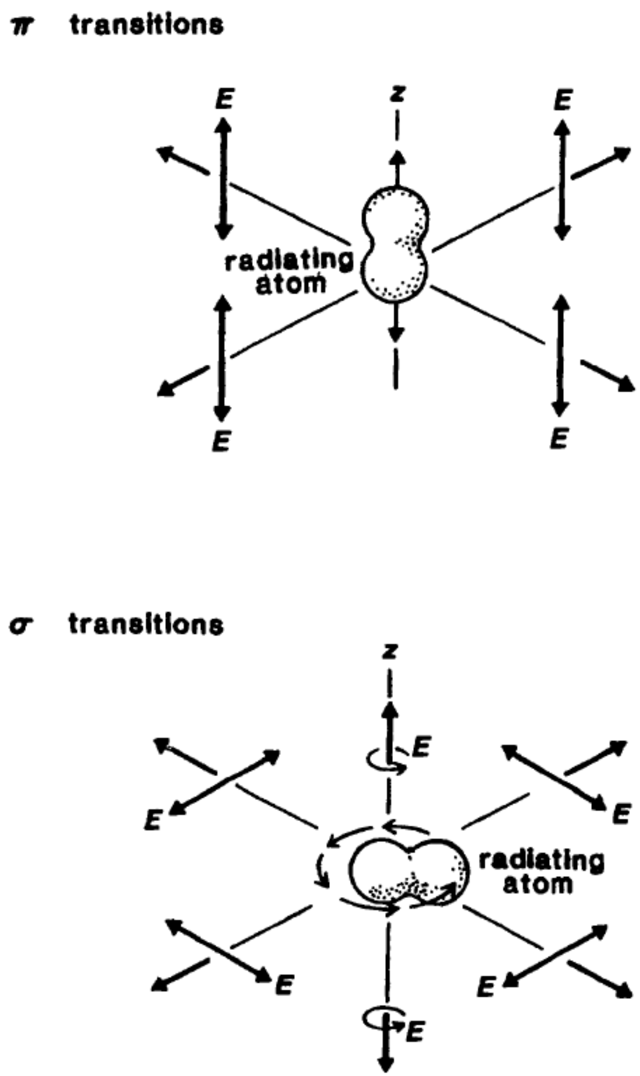
LET'S RECAP...

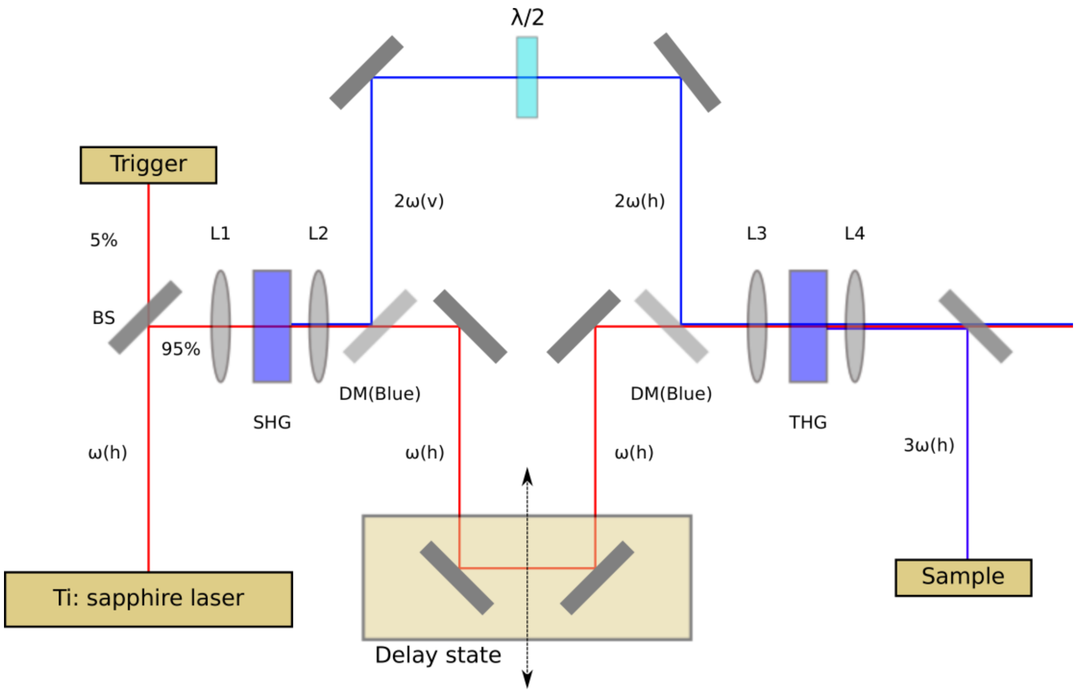
Number 5


ε
2
1
$\hbar\omega_a$
m
n
ε
Greatly expanded
energy scale

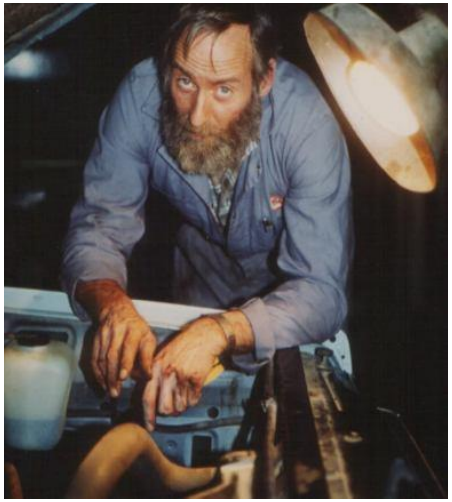

E_2
m = 1
2
3
$g_2 = 3$
E_1
n = 1
2
$g_1 = 2$

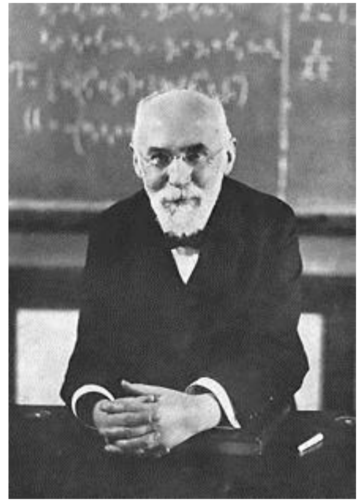
Number 4

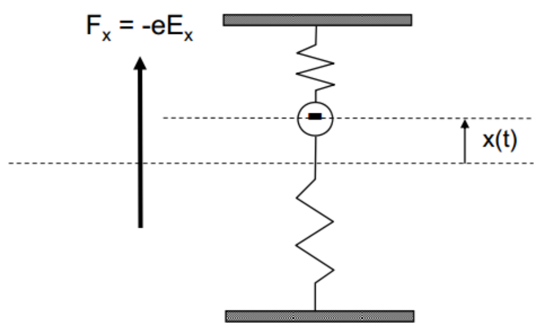

π transitions
E
z
E
radiating atom
E
E
σ transitions
z
E
E
E
radiating atom
E
E
E

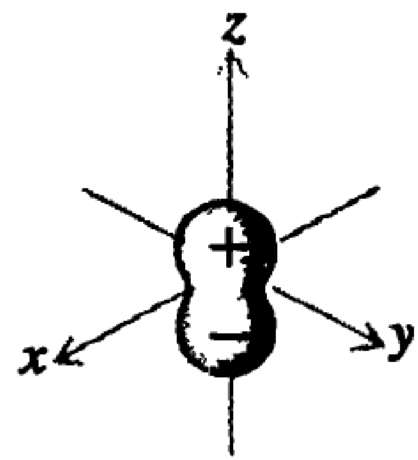

λ/2
Trigger
2ω(v)
2ω(h)
5%
L1
L2
L3
L4
BS
95%
DM(Blue)
DM(Blue)
SHG
THG
ω(h)
ω(h)
ω(h)
3ω(h)
Ti: sapphire laser
Delay state
Sample

Number 3

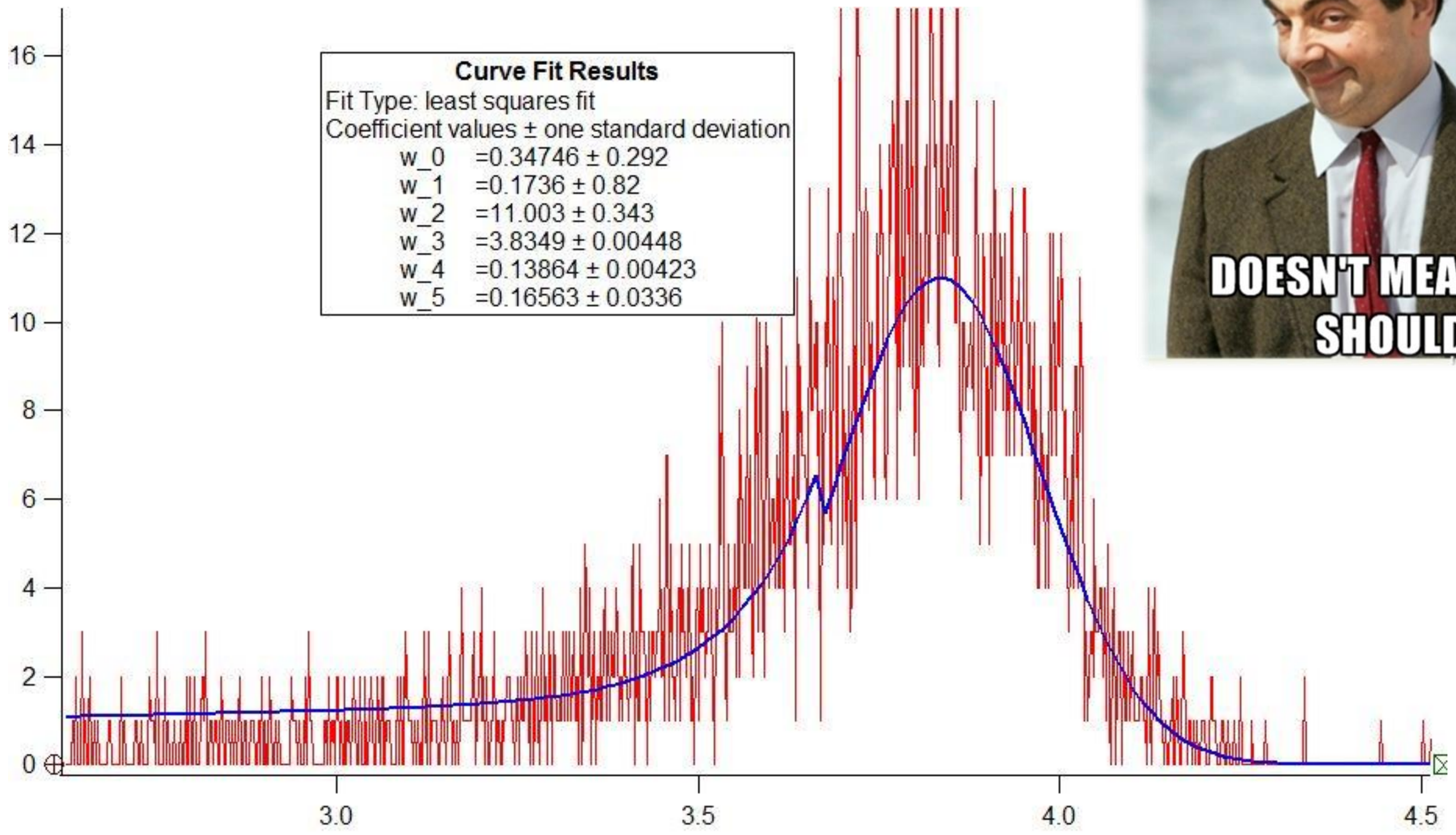






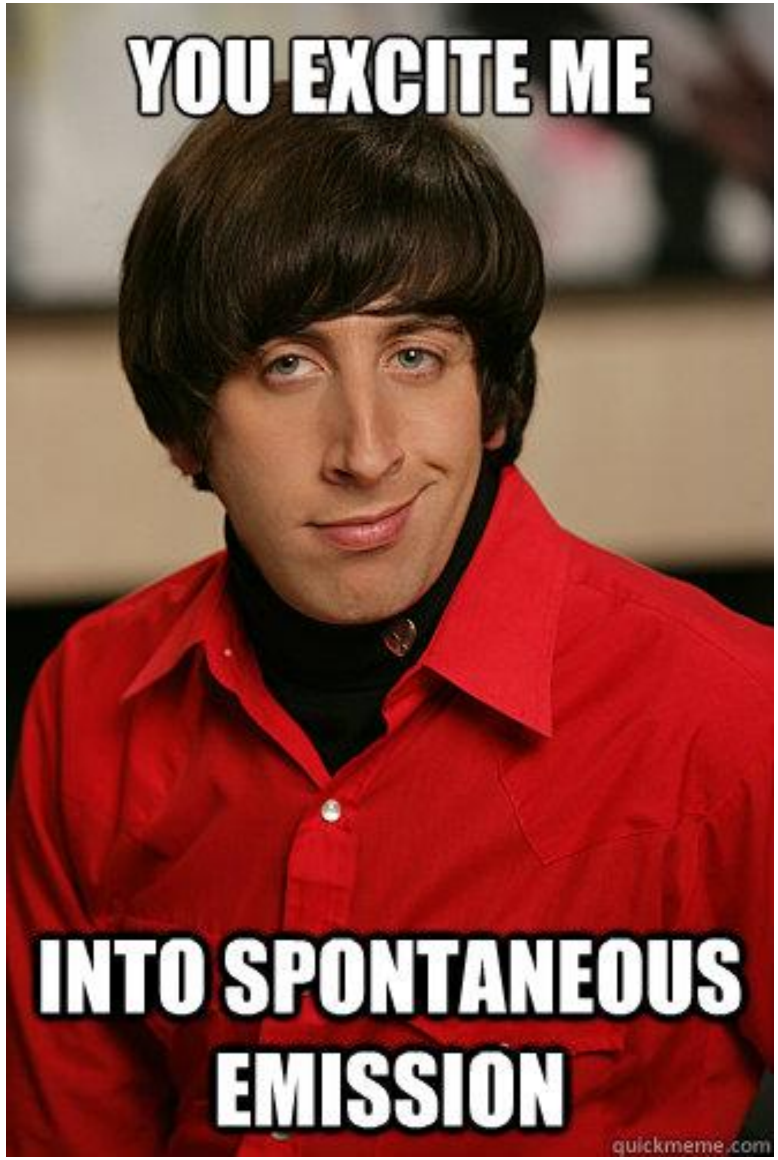


Number 2


Curve Fit Results
Fit Type: least squares fit
Coefficient values ± one standard deviation
w_0 =0.34746 ± 0.292
w_1 =0.1736 ± 0.82
w_2 =11.003 ± 0.343
w_3 =3.8349 ± 0.00448
w_4 =0.13864 ± 0.00423
w_5 =0.16563 ± 0.0336
16
14
12
10
8
6
4
2
0
3.0
3.5
4.0
4.5


JUST BECAUSE YOU CAN
DOESN'T MEAN YOU SHOULD
memegenerator.net

Number 1



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