1.3. If levels 1 and 2 in Fig. 1.1 are separated by an energy $E_2 - E_1$ such that the corresponding transition frequency falls in the middle of the visible range, calculate the ratio of the populations of the two levels in thermal equilibrium at room temperature.

1.4. In thermal equilibrium at $T = 300$ K, the ratio of level populations N_2/N_1 for some particular pair of levels is given by $1/e$. Calculate the frequency ν for this transition. In what region of the em spectrum does this frequency fall?

1.5. A laser cavity consists of two mirrors with reflectivities $R_1 = 1$ and $R_2 = 0.5$, while the internal loss per pass is $L_i = 1\%$. Calculate total logarithmic losses per pass. If the length of the active material is $l = 7.5$ cm and the transition cross section is $\sigma = 2.8 \times 10^{-19}$ cm^2, calculate the threshold inversion.

1.6. The beam from a ruby laser ($\lambda \cong 694$ nm) is sent to the moon after passing through a telescope of 1-m diameter. Calculate the approximate value of beam diameter on the moon assuming that the beam has perfect spatial coherence. (The distance between earth and moon is approximately 384,000 km.)

1.7. The brightness of probably the brightest lamp so far available (PEK Labs type 107/109^{TM}, excited by 100 W of electrical power) is about 95 W/cm^{2}sr in its most intense green line ($\lambda = 546$ nm). Compare this brightness with that of a 1-W argon laser ($\lambda = 514.5$ nm), which can be assumed to be diffraction-limited.

2.3. For blackbody radiation find the maximum of ρ_λ versus λ. Show in this way that the wavelength λ_M at which the maximum occurs satisfies the relationship $\lambda_M T = hc/ky$ (Wien's law), where the quantity y satisfies the equation $5[1 - \exp(-y)] = y$. From this equation find an approximate value of y.

2.7. The neon laser transition at $\lambda = 1.15$ μm is predominantly Doppler broadened to $\Delta\nu_0^* = 9 \times 10^8$ Hz. The upper state lifetime is $\approx 10^{-7}$ s. Calculate the peak cross section assuming that the laser transition lifetime is equal to the upper state lifetime.

Example 2.1. *Estimate of τ_{sp} and A for electric-dipole-allowed-and-forbidden transitions.* For an electric-dipole-allowed transition at a frequency corresponding to the middle of the visible range, an estimate on the order of magnitude of A is obtained from Eq. (2.3.19) by substituting the values $\lambda = c/\nu = 500$ nm and $|\mu| \approx ea$, where a is the atomic radius ($a \cong 0.1$ nm). We therefore obtain $A \cong 10^8$ s^{-1} (i.e., $\tau_{sp} \cong 10$ ns). For magnetic dipole transitions A is approximately 10^5 times smaller, and therefore $\tau_{sp} \approx 1$ ms. Note: According to Eq. (2.3.19), A increases as the cube of the frequency, so that the importance of spontaneous emmission increases rapidly with frequency. In fact spontaneous emission is often negligible in the middle- to far-infrared where nonradiative decay usually dominates. On the other hand when we consider the x-ray region (say, $\lambda \leq 5$ nm), τ_{sp} becomes exceedingly short (10–100 fs), which constitutes a major problem for achieving a population inversion in x-ray lasers.

Example 2.4. *Natural linewidth of an allowed transition.* As a representative example, we can find an order of magnitude estimate for $\Delta\nu_{nat}$ for an electric-dipole-allowed transition. Assuming $|\nu| = ea$ with $a \cong 0.1$ nm and $\lambda = 500$ nm (green light), we found in Example 2.1 that $\tau_{sp} \cong 10$ ns. From Eq. (2.5.13) we then obtain $\Delta\nu_{nat} \cong 16$ MHz. Note that $\Delta\nu_{nat}$, just as $A = 1/\tau_{sp}$, is expected to increase with frequency as ν_0^3. Therefore natural linewidth increases very rapidly for transitions at shorter wavelengths (to the uv or x-ray region).