# Scope of the Lecture

1. Laser amplifiers
2. Chirped pulse amplification
3. Nonlinear material response
4. Second harmonic generation
5. Optical parametric generation
6. Optical parametric oscillators

Reading: Ch. 12

# Amplifier saturation

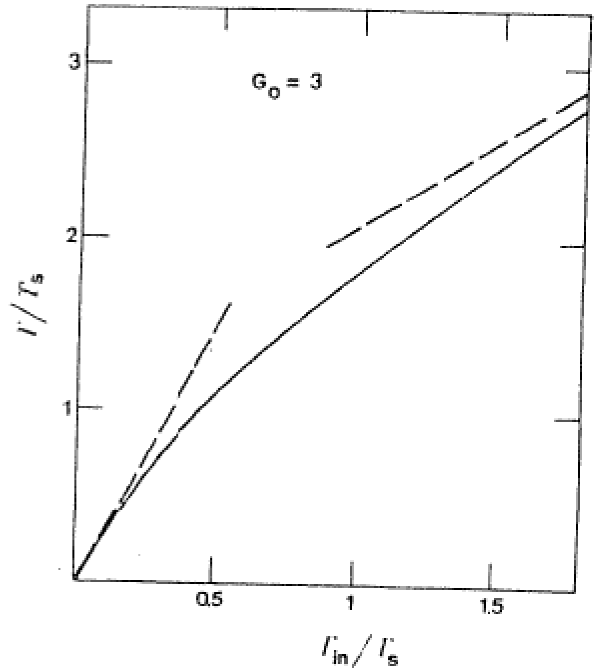
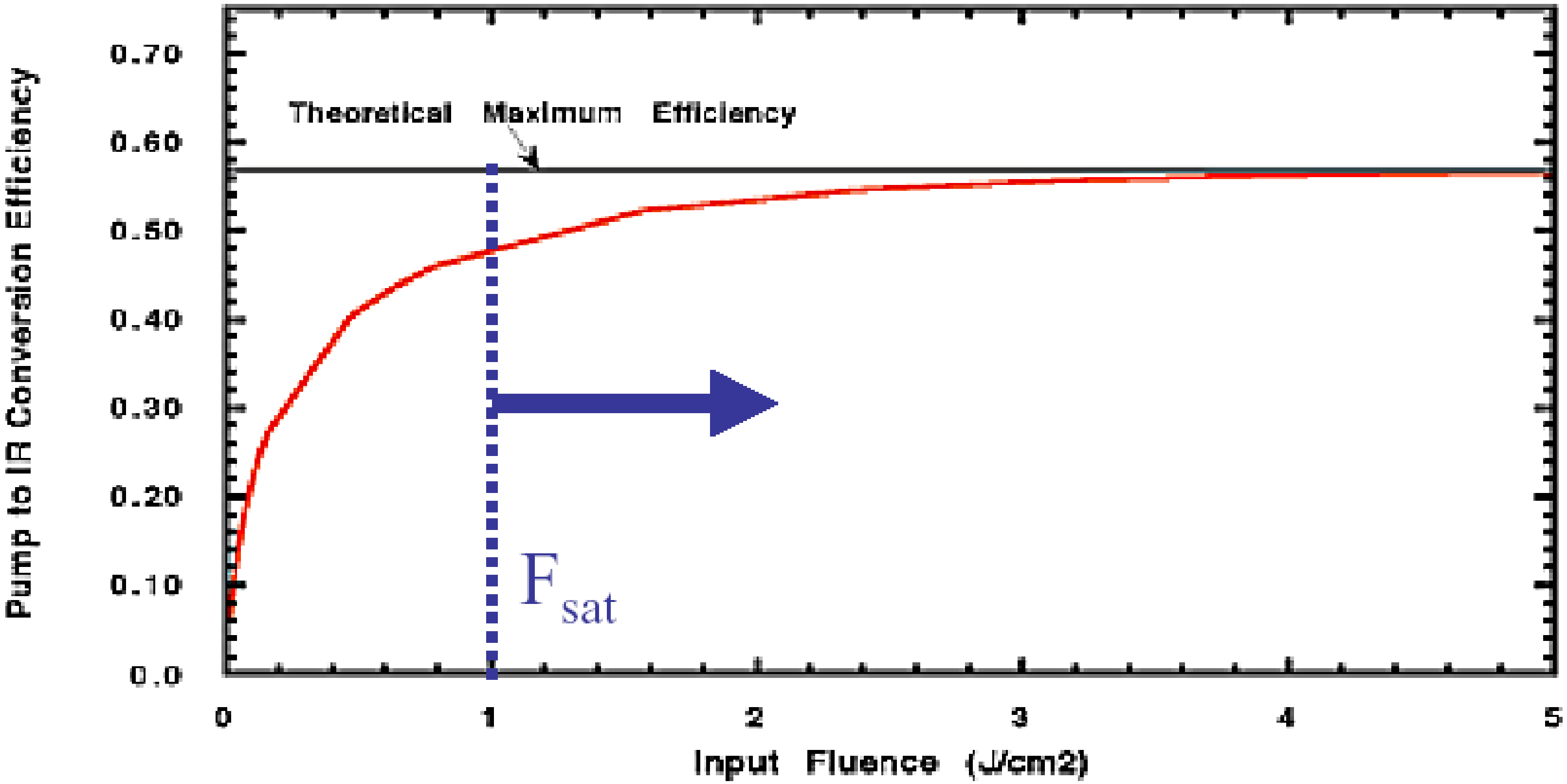



*FIG. 12.2.* Output laser energy fluence $\Gamma$ versus input fluence $\Gamma_{in}$ for a laser amplifier with a small signal gain $G_0 = 3$. Energy fluence is normalized to the laser saturation fluence $\Gamma_s = h\nu/\sigma$.

# Amplifier saturation

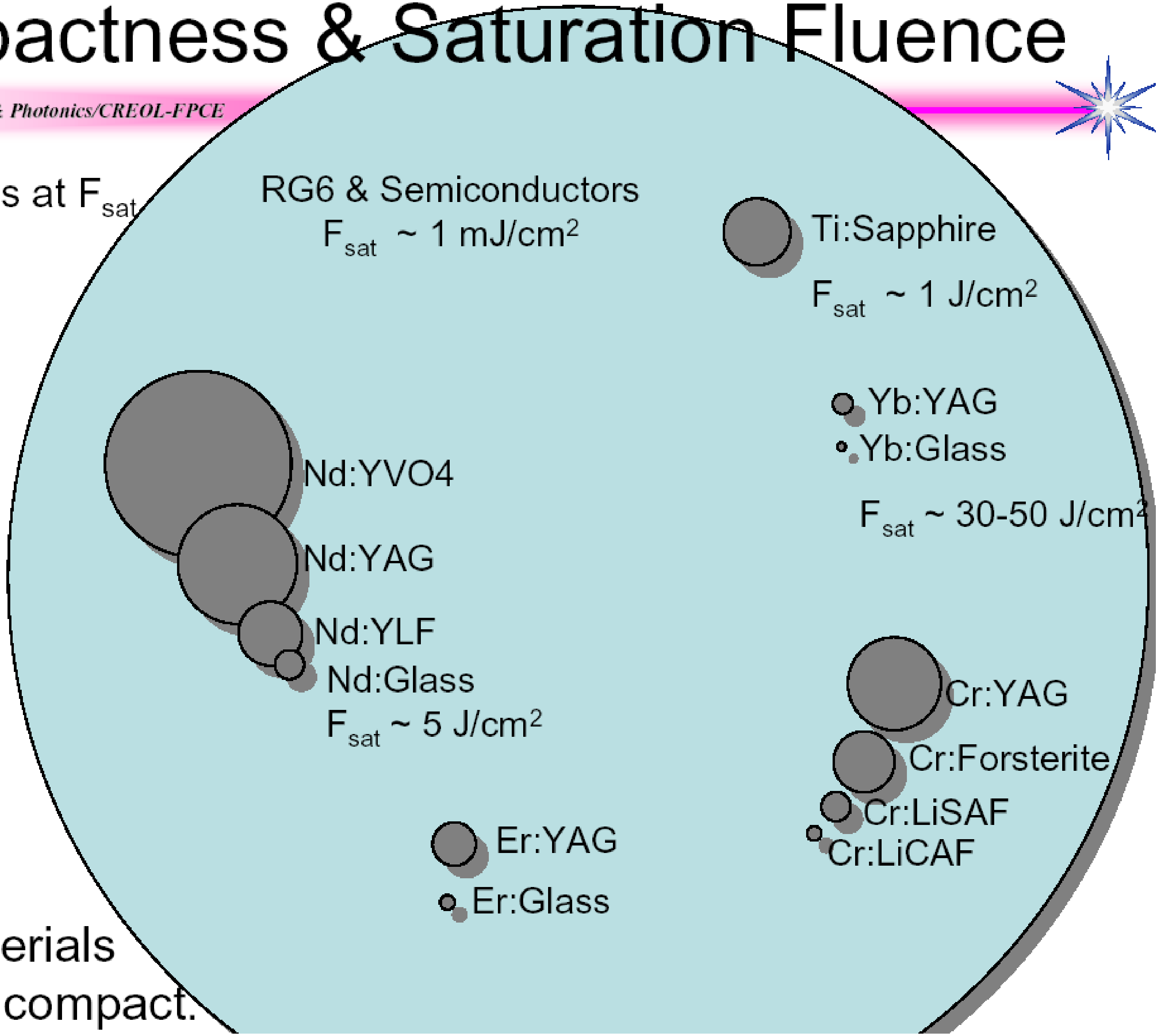
Operate amplifier above saturation for max efficiency



# Compactness & Saturation Fluence

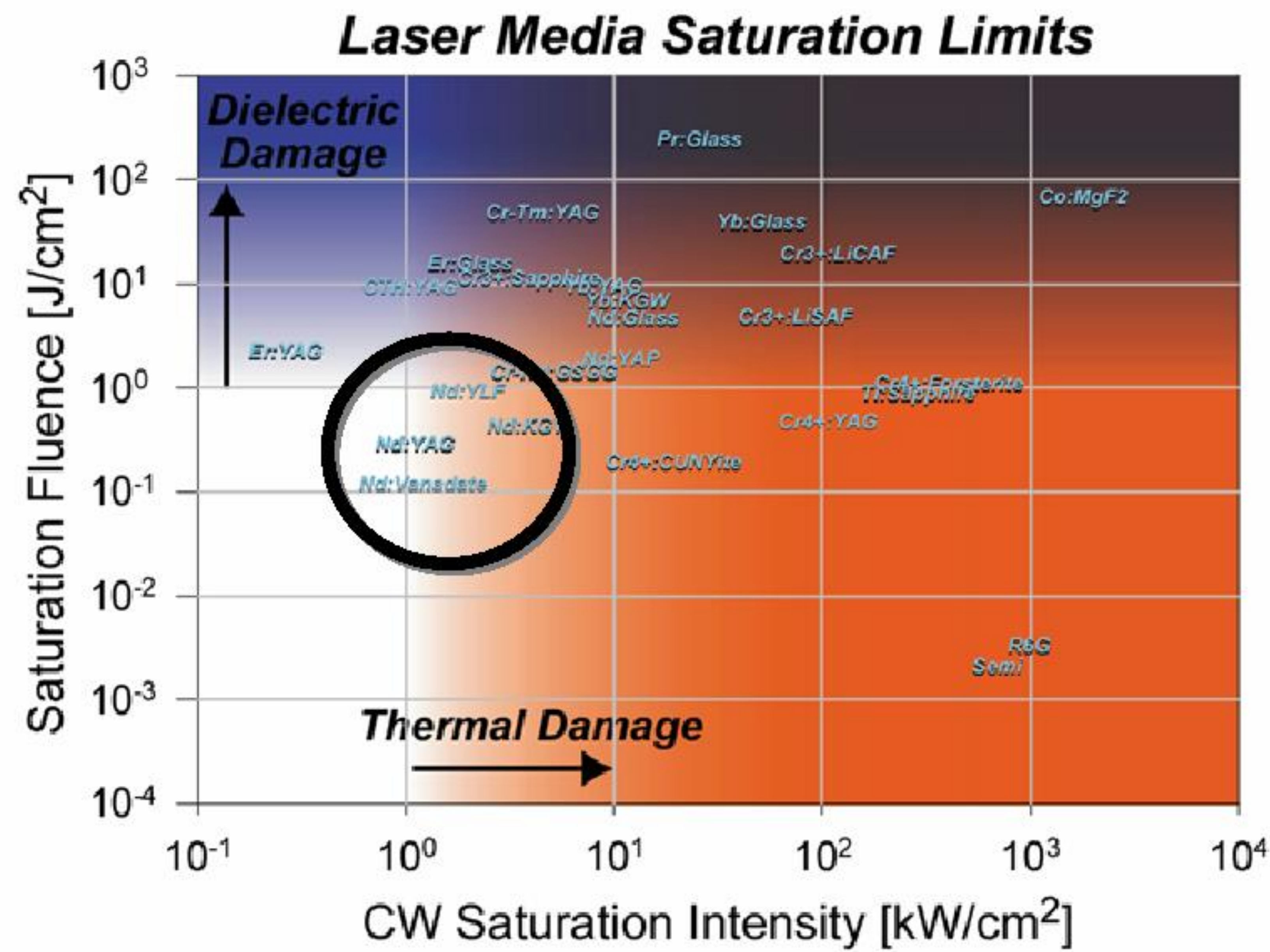
UCF College of Optics & Photonics/CREOL-FPCE

1-J Beam Areas at $F_{sat}$



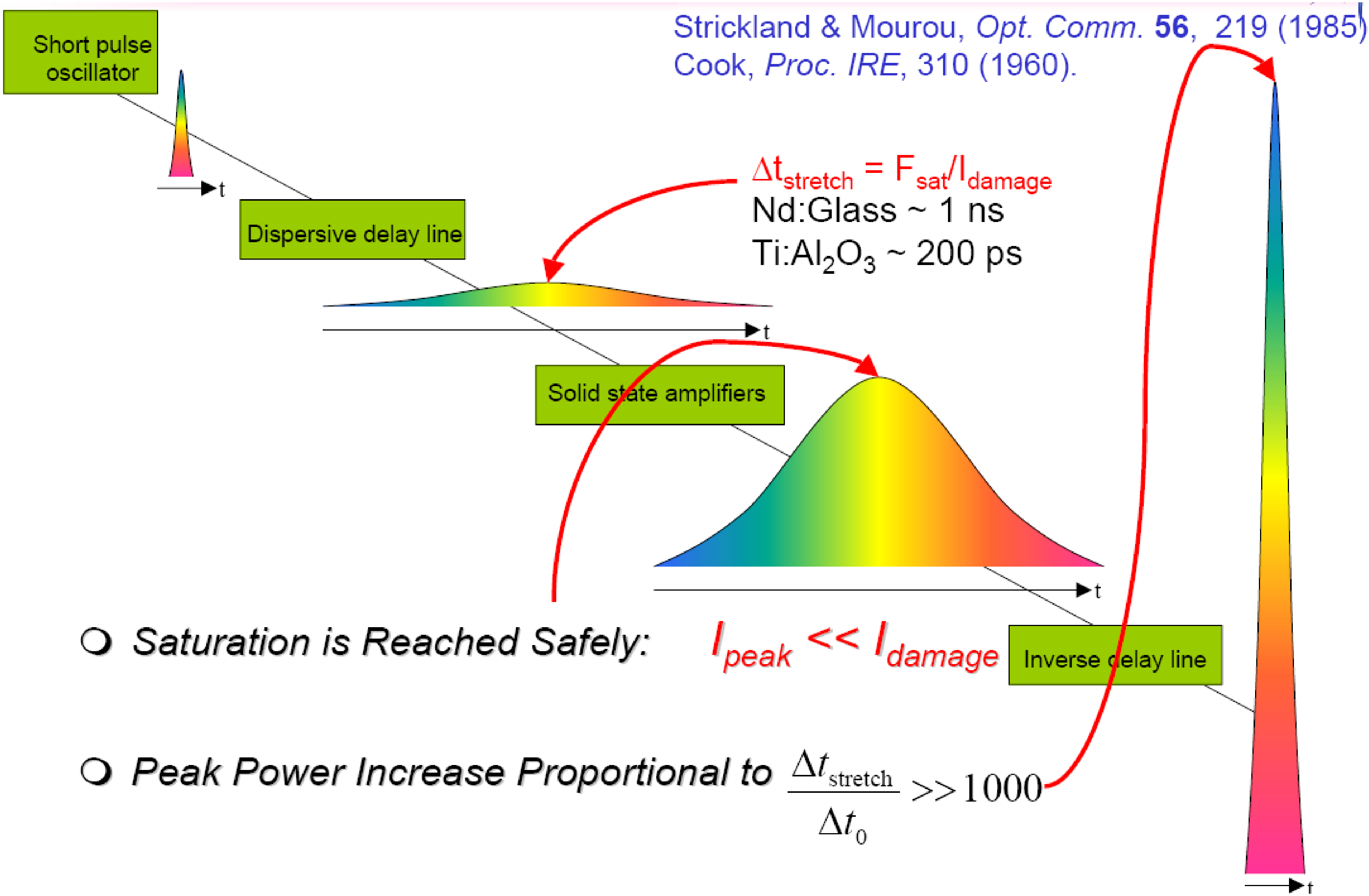

High $F_{sat}$ materials can be more compact.

# High-peak versus high-average power



# Chirped-pulse amplification (CPA)

Strickland & Mourou, *Opt. Comm.* **56**, 219 (1985).
Cook, *Proc. IRE*, 310 (1960).




- *Saturation is Reached Safely:* $I_{peak} << I_{damage}$
- *Peak Power Increase Proportional to* $\frac{\Delta t_{stretch}}{\Delta t_0} >> 1000$

# Dispersive pulse stretcher

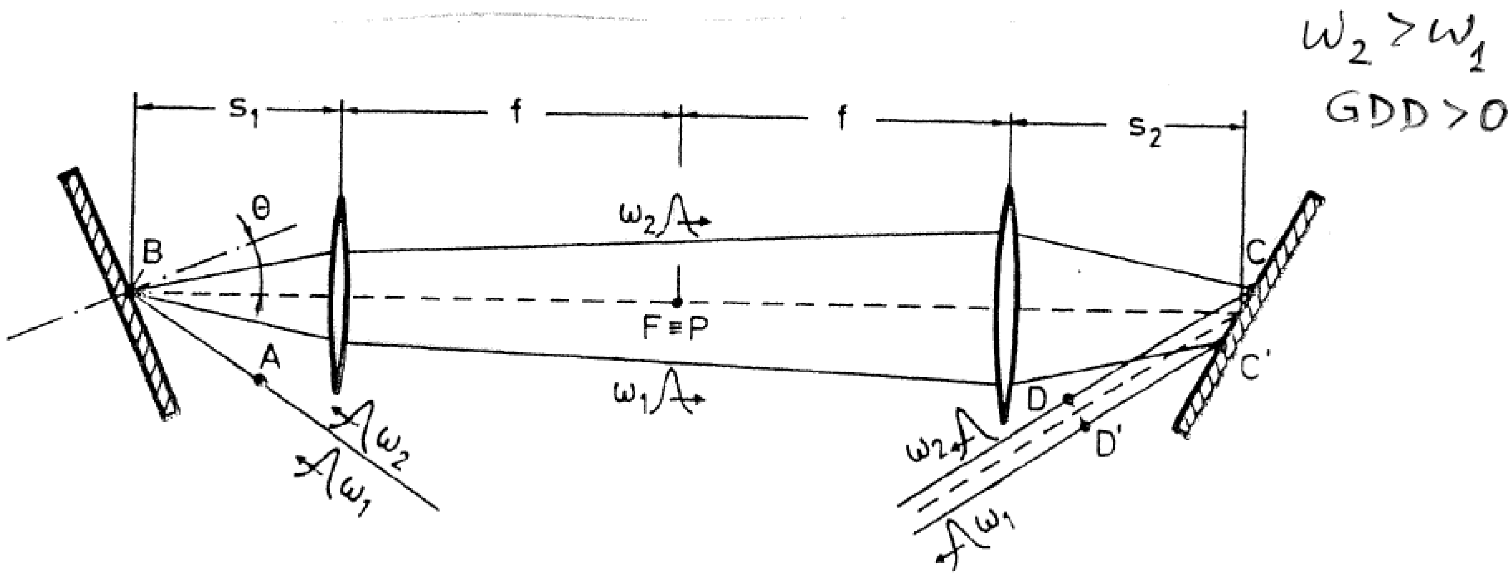



*FIG. 12.15.* Pulse expander consisting of two gratings, in an antiparallel configuration, with a 1-to-1 inverting telescope between them.

# Dispersive pulse compressor

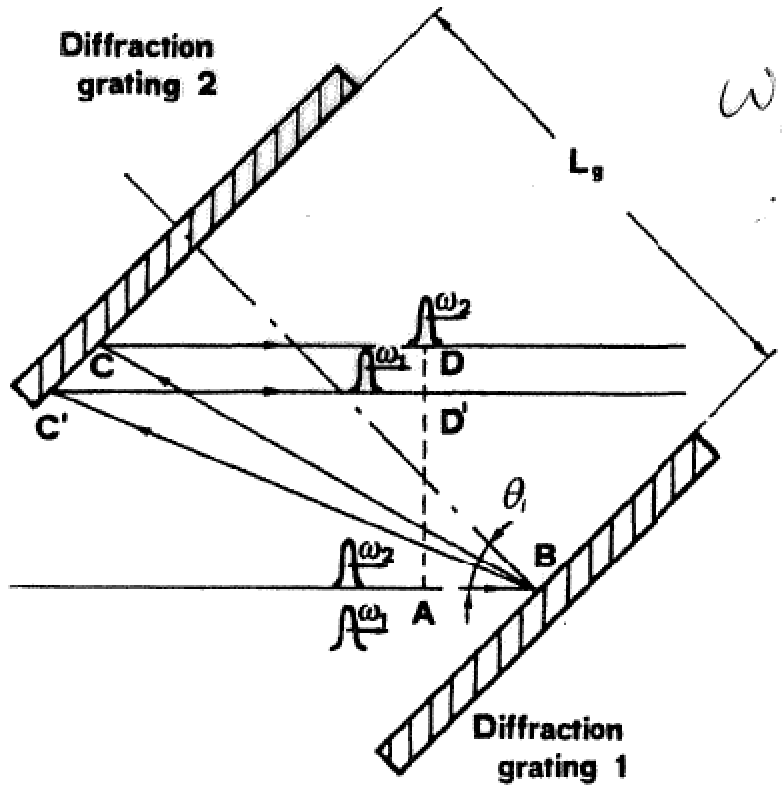
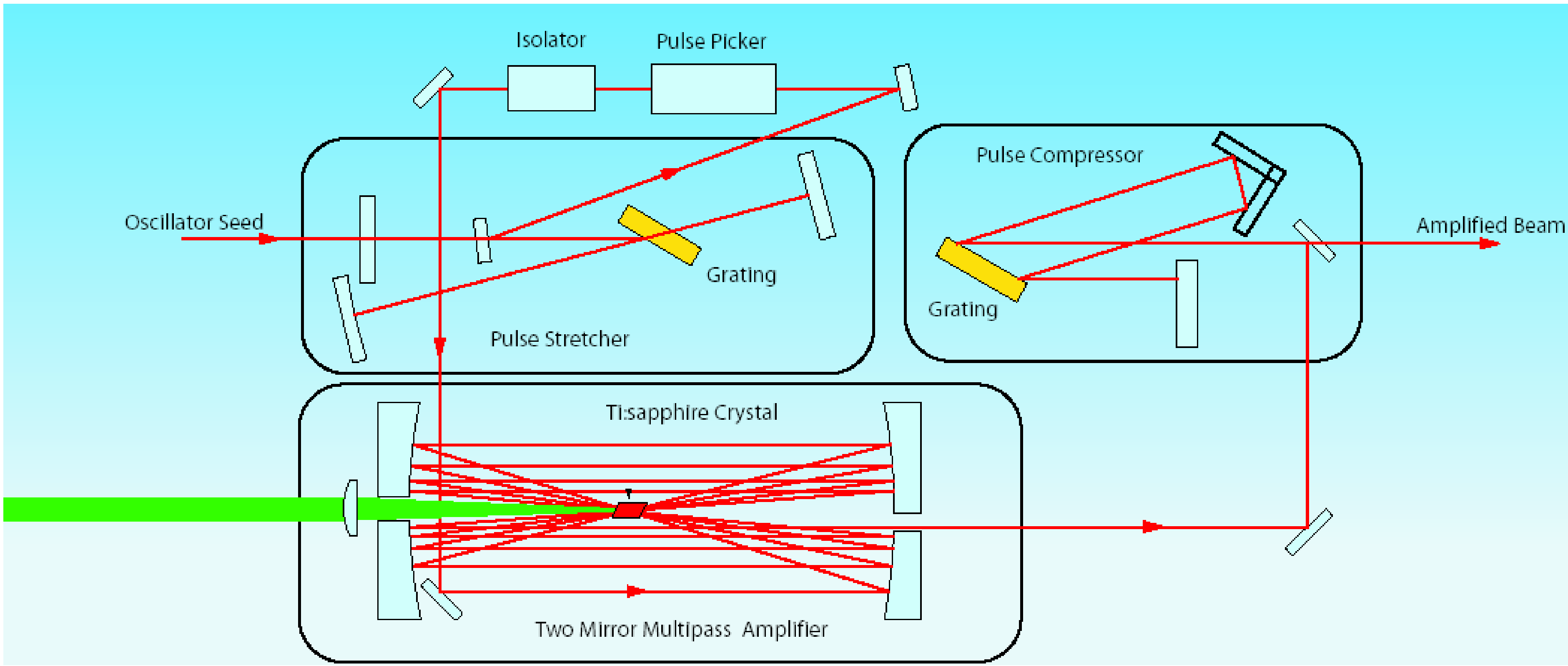



*FIG. 12.14.* Grating-pair for pulse compression.

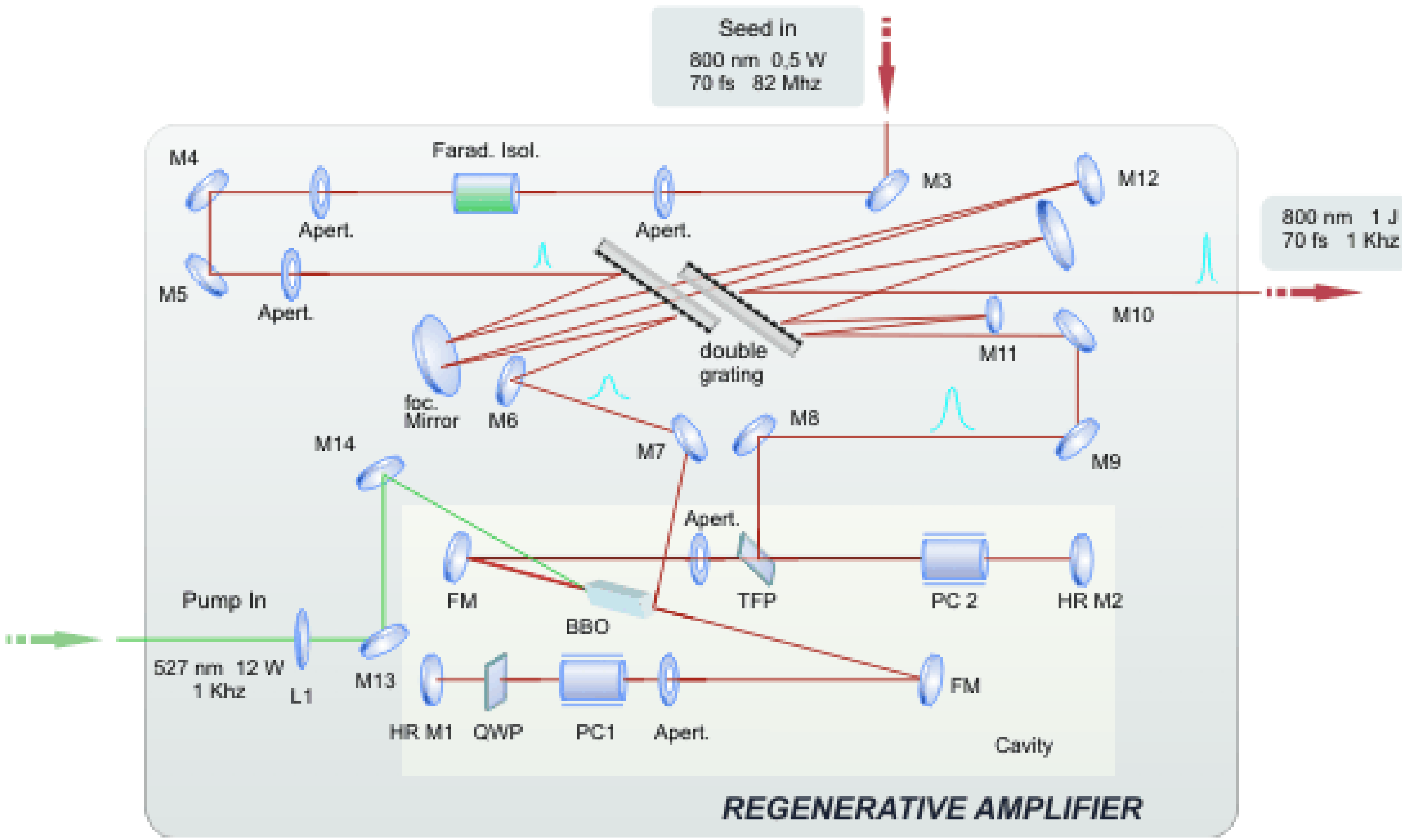
$\omega_2 > \omega_1$

GDD < 0

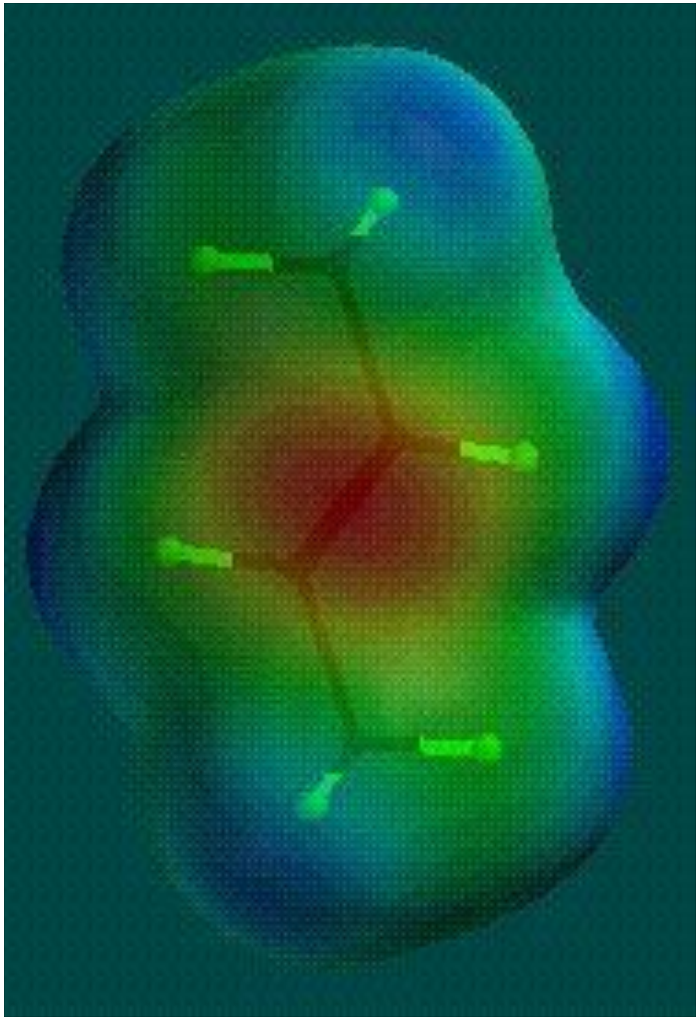
# Multipass amplifier



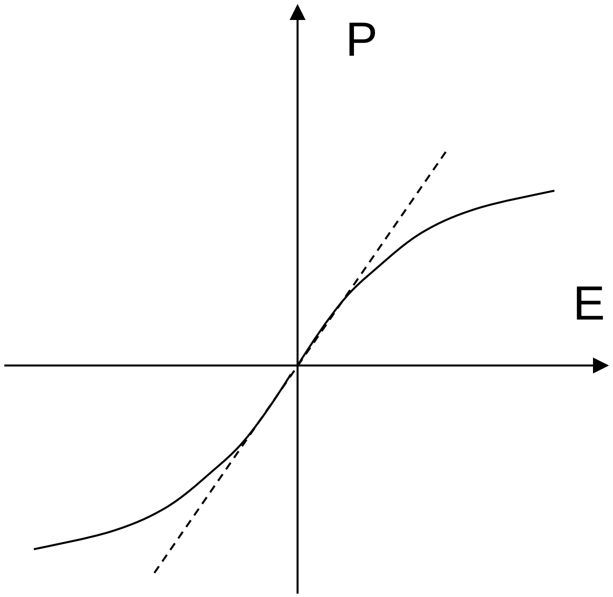
# Regenerative amplifier



# Nonlinear polarization



$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$



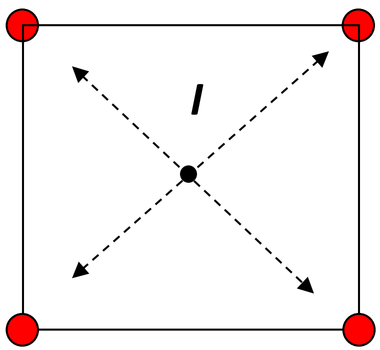

$$P = P_0 + \varepsilon_0 \left( \chi^{(1)} E + \chi^{(2)} EE + \chi^{(3)} EEE + \ldots \right)$$

# Inversion symmetry and second-order processes

In materials with inversion symmetry 2nd order nonlinear processes are absent:

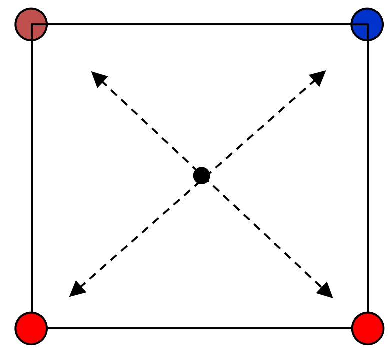
$$\chi_{ijk}^{(2)} \equiv 0$$

Inversion center ***I*** exists




Symmetry classes:
(432).
(622).
(422).
Isotropic solids.
Atomic gasses.
Molecular gasses.

Inversion center ***I*** absent



All remaining symmetry classes.

Surfaces of isotropic media.

Formal proof by applying inversion operator ***I*** :

$$\mathbf{P}^{(2)} = \varepsilon_0 \chi^{(2)} : \mathbf{EE}$$

$$I(\mathbf{P}^{(2)}) = \varepsilon_0 I(\chi^{(2)}) : I(\mathbf{E}) I(\mathbf{E})$$

$$-\mathbf{P}^{(2)} = \varepsilon_0 I(\chi^{(2)}) : (-\mathbf{E})(-\mathbf{E}) \Rightarrow I(\chi^{(2)}) = \chi^{(2)} \Rightarrow \chi^{(2)} \equiv 0. \quad (3.4)$$

# Parametric frequency conversion

Consider only second-order interactions:

$$\mathbf{P}_{NL}(\omega_1) \propto \boldsymbol{\chi}^{(2)}(\omega_1;\omega_2,\omega_3):\mathbf{E}(\omega_2)\mathbf{E}(\omega_3)$$

Processes:

• Parametric upconversion:

Sum-frequency mixing (SFM) $\omega_1 = \omega_2 + \omega_3, \omega_2 \neq \omega_3$

Second harmonic generation (SHG) $\omega_1 = \omega_2 + \omega_3, \omega_2 = \omega_3$

Electro-optic modulation $\omega_1 = \omega_2 + \omega_3, \omega_2 = 0$

• Parametric downconversion:

Difference-frequency mixing (DFM) $\omega_2 = \omega_1 - \omega_3, \omega_2 \neq \omega_3$

Parametric generation $\omega_2 = \omega_1 - \omega_3, \omega_2 \neq \omega_3$

Optical rectification $\omega_2 = \omega_1 - \omega_3, \omega_1 = \omega_3$

***Particular process is selected by momentum conservation***

$$\Delta\vec{k} = \vec{k}_1 - \vec{k}_2 - \vec{k}_3 = 0 \qquad k_j = \frac{2\pi n_j}{\lambda_j}$$

# Coupled wave equations

Consider sum-frequency mixing process: $\omega_1 = \omega_2 + \omega_3;$ described by susceptibility $\chi^{(2)}_{\mu\alpha_1\alpha_2}(-\omega_1;\omega_2,\omega_3)$.

From Maxwell's equation, the nonlinear wave equation for $\omega_1$ field:

$$\frac{\partial E_1}{\partial z}\exp(i(\omega_1 t - k_1 z)) = i\frac{\omega_1 d_{eff}}{cn_1}E_2\exp(i(\omega_2 t - k_2 z))E_3\exp(i(\omega_3 t - k_3 z)). \quad (3.29)$$

Need to solve coupled wave nonlinear equations for all fields:

$$\frac{\partial E_1}{\partial z} = i\frac{\omega_1 d_{eff}}{cn_1}E_2E_3\exp(i\Delta kz)). \qquad \omega_1 = \omega_2 + \omega_3;$$

Wave-vector mismatch

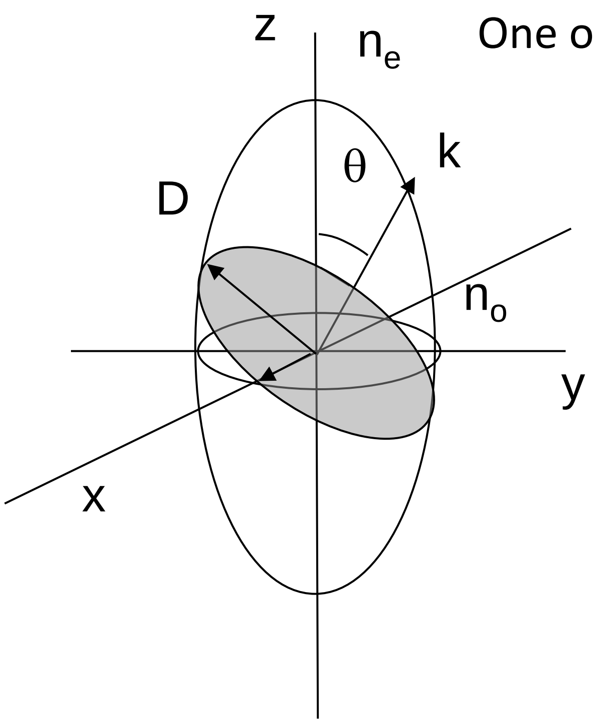
$$\Delta k = k_1 - k_2 - k_3$$

$$\frac{\partial E_2}{\partial z} = i\frac{\omega_2 d_{eff}}{cn_2}E_1E_3^*\exp(-i\Delta kz)). \qquad \omega_2 = \omega_1 - \omega_3;$$

$$\frac{\partial E_3}{\partial z} = i\frac{\omega_3 d_{eff}}{cn_3}E_1E_2^*\exp(-i\Delta kz)). \qquad \omega_3 = \omega_1 - \omega_2;$$

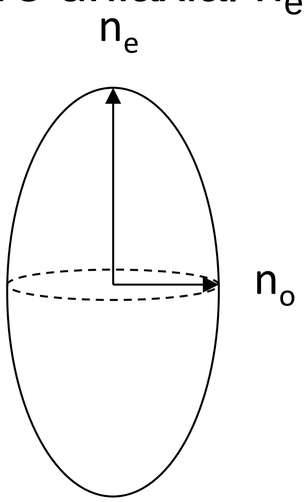
# Birefringent uniaxial crystals

One optical axis || z

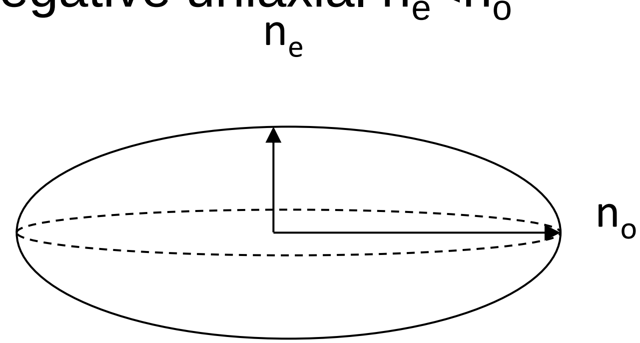



$$\frac{1}{[n_e(\theta,\omega_i)]^2} = \frac{\cos^2\theta}{[n_o(0,\omega_i)]^2} + \frac{\sin^2\theta}{[n_e(\pi/2,\omega_i)]^2}$$

Positive uniaxial $n_e > n_o$

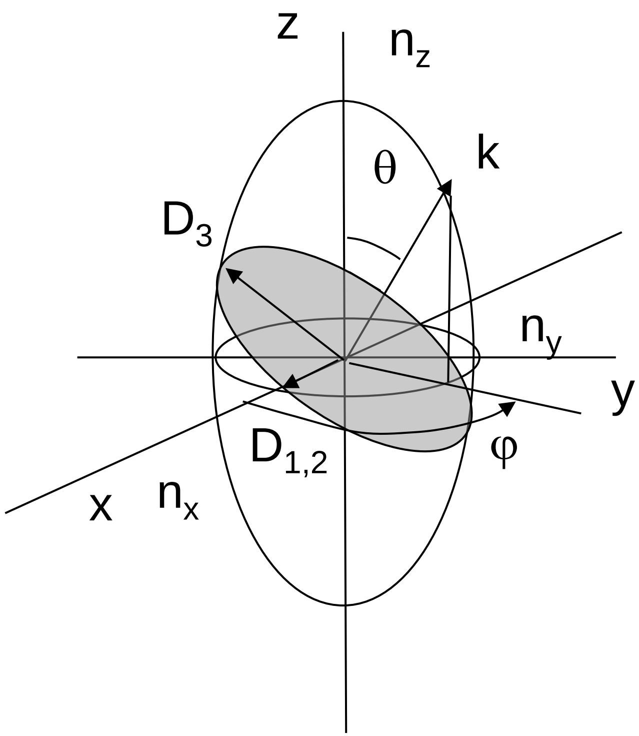



Negative uniaxial $n_e < n_o$



# Birefringent biaxial crystals

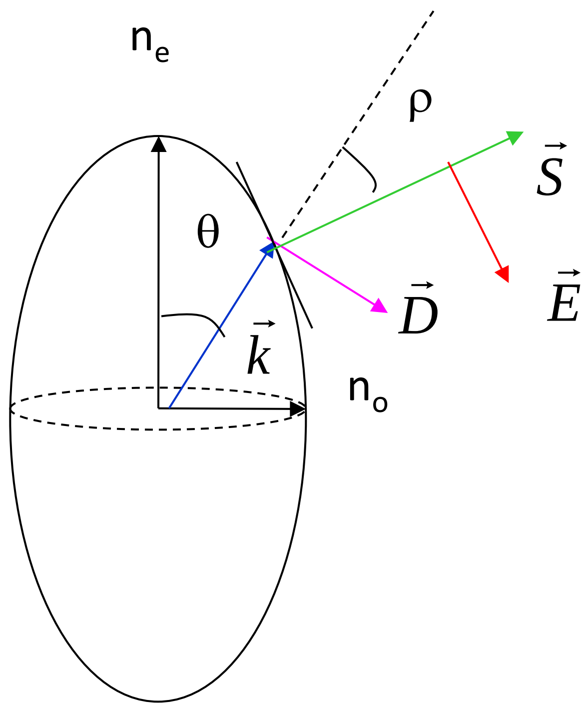
$$n_x \neq n_y \neq n_z$$




Two optical axes

$$\frac{\sin^2\theta\cos^2\varphi}{n_i^{-2}-n_{x,i}^{-2}}+\frac{\sin^2\theta\sin^2\varphi}{n_i^{-2}-n_{y,i}^{-2}}+\frac{\cos^2\theta}{n_i^{-2}-n_{z,i}^{-2}}=0$$

# Waves in birefringent crystals




$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

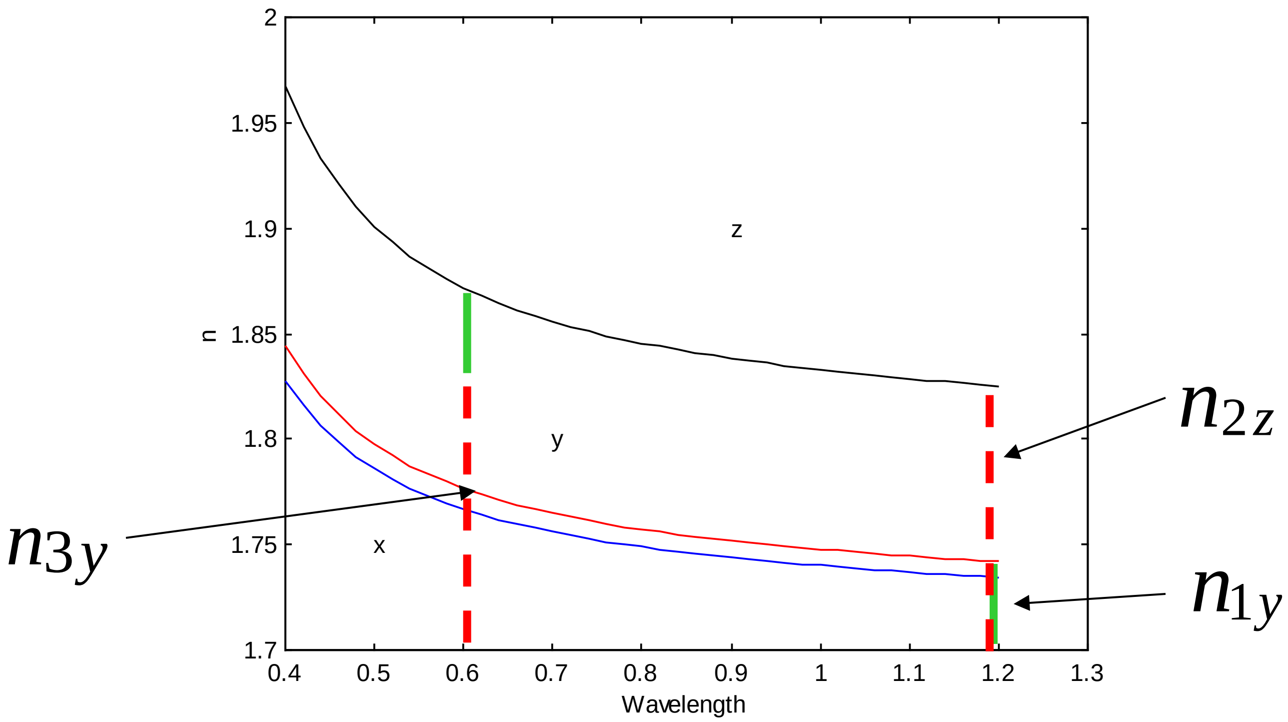
$$Wavevector\ \vec{k} \perp \vec{D}$$

$$Poynting\,vector\ \vec{S} \perp \vec{E}$$

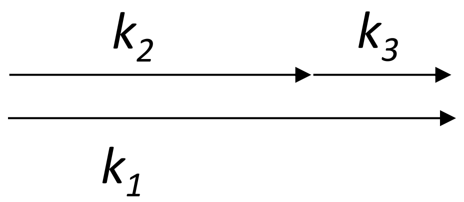
$$Poynting\,vector\ walkoff\ \rho = -\frac{1}{n}\frac{dn}{d\theta}$$

# Birefringence phase-matching

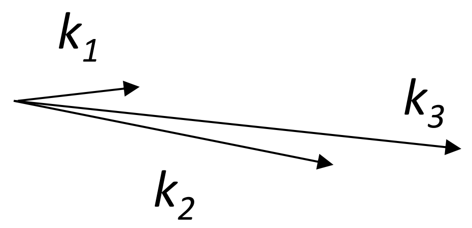
$$\omega_1 + \omega_2 = \omega_3, \; \omega_1 = \omega_2$$




**Collinear**



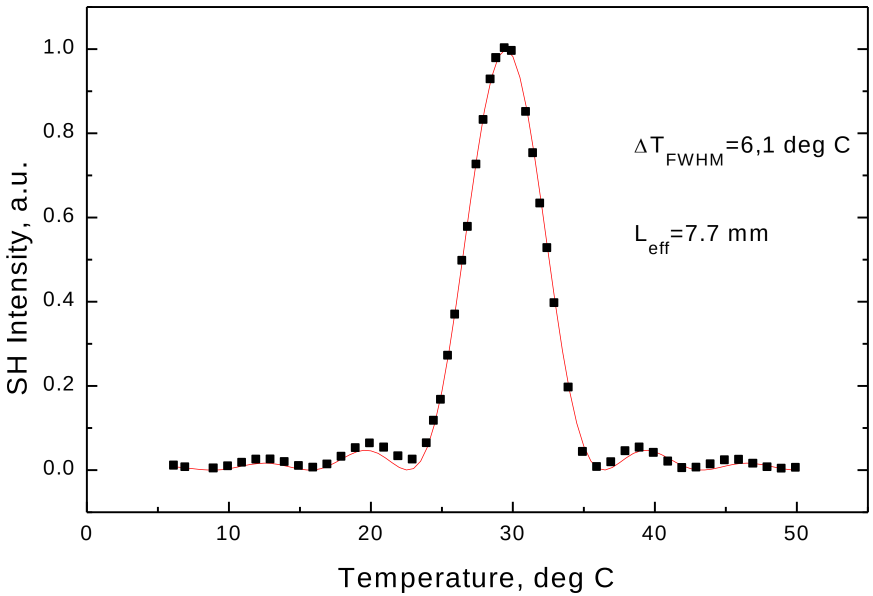

**Noncollinear**




$$\frac{n_{1y}}{1.2\mu m} + \frac{n_{2z}}{1.2\mu m} = \frac{n_{3y}}{0.6\mu m} \Rightarrow n_{1y} + n_{2z} = 2n_{3y}$$

# Momentum conservation

$$\omega_3 = \omega_1 + \omega_2 \qquad \Delta\vec{k} = \vec{k}_3 - \vec{k}_2 - \vec{k}_1 = 0$$




$$I_3(L) = \frac{\omega_3^2 d_{eff}^2}{2\varepsilon_0 c^3 n_1 n_2 n_3} I_1 I_2 L^2 \,\mathrm{sinc}^2(\Delta k L/2)$$

- FWHM Phase-matching (acceptance) bandwidth: $\frac{\Delta k L}{2} \approx 0.9\pi$

- Acceptance bandwidth with respect to external parameter $\psi$: $\psi : \lambda, T, \theta$

$$\delta\psi = \frac{5.57}{L}\left|\frac{\partial \Delta k}{\partial \psi}\right|^{-1}$$

# CWE solutions: SHG

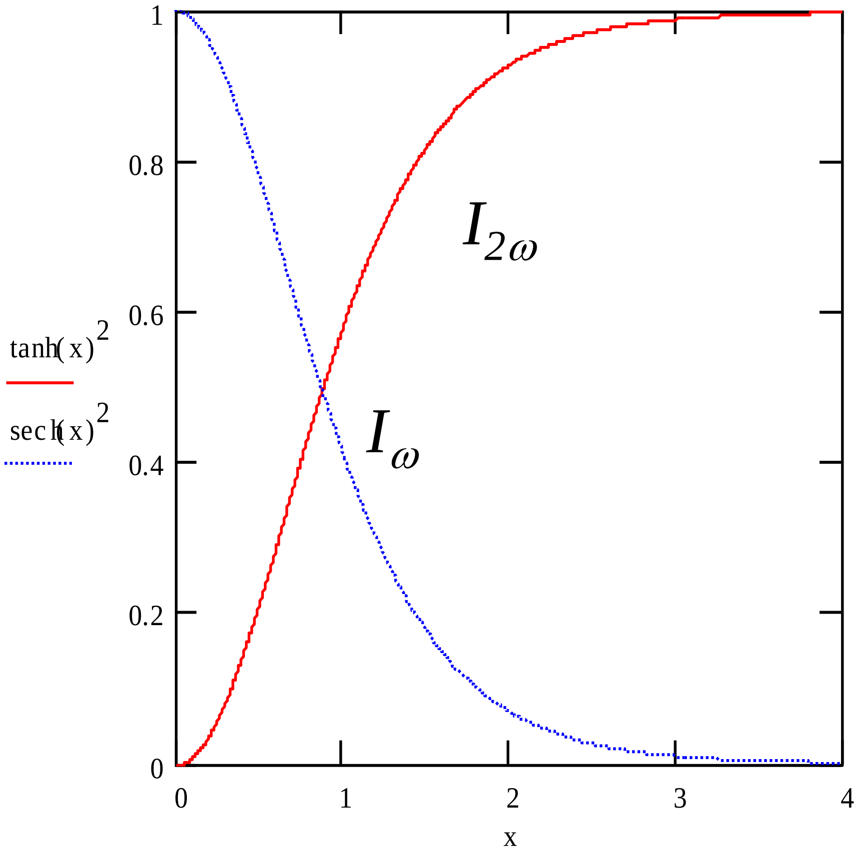
$$I_{2\omega}(L) = I_{\omega}(0)\tanh^2(\Gamma L),$$

$$I_{\omega}(L) = I_{\omega}(0)\,\mathrm{sech}^2(\Gamma L).$$

where $$\Gamma = \frac{4\pi d_{eff}\sqrt{I_{\omega}(0)}}{\sqrt{2cn_{\omega}^2 n_{2\omega}\varepsilon_0\lambda_{\omega}^2}}.$$

Conversion efficiency :

$$\eta(L) = I_{2\omega}(L)/I_{\omega}(0) = \tanh^2(\Gamma L)$$



# Sum-frequency mixing. Application example

Laser guide-star
Excitation of Na fluorescence with 589 nm beam forcused 90 km above the Earth.
*Boeing, AFRL.*

# CWE solutions: OPG

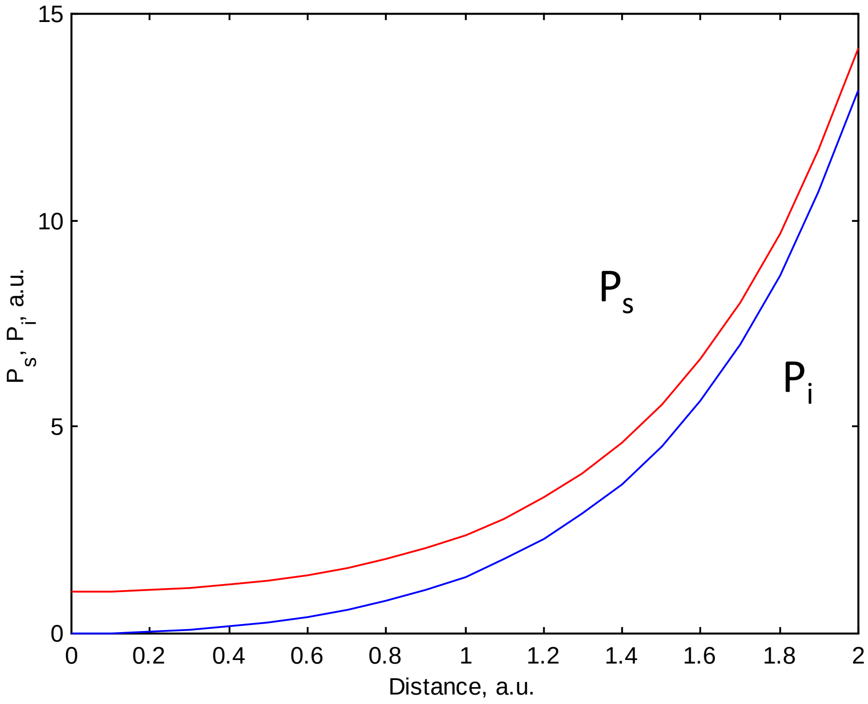
Process: $\omega_i = \omega_p - \omega_s$

p – pump, s – signal, i - idler

$$P_S(L)/P_S(0) = \cosh^2(L/L_{NL}),$$
$$P_i(L)/P_i(0) = \sinh^2(L/L_{NL}).$$

where: $$L_{NL} = \frac{1}{4\pi d_{eff}} \sqrt{\frac{2\varepsilon_0 n_p n_s n_i c \lambda_s \lambda_i}{I_p(0)}},$$

- *Exponential growth* of signal and idler.
- Used as gain in parametric devices.

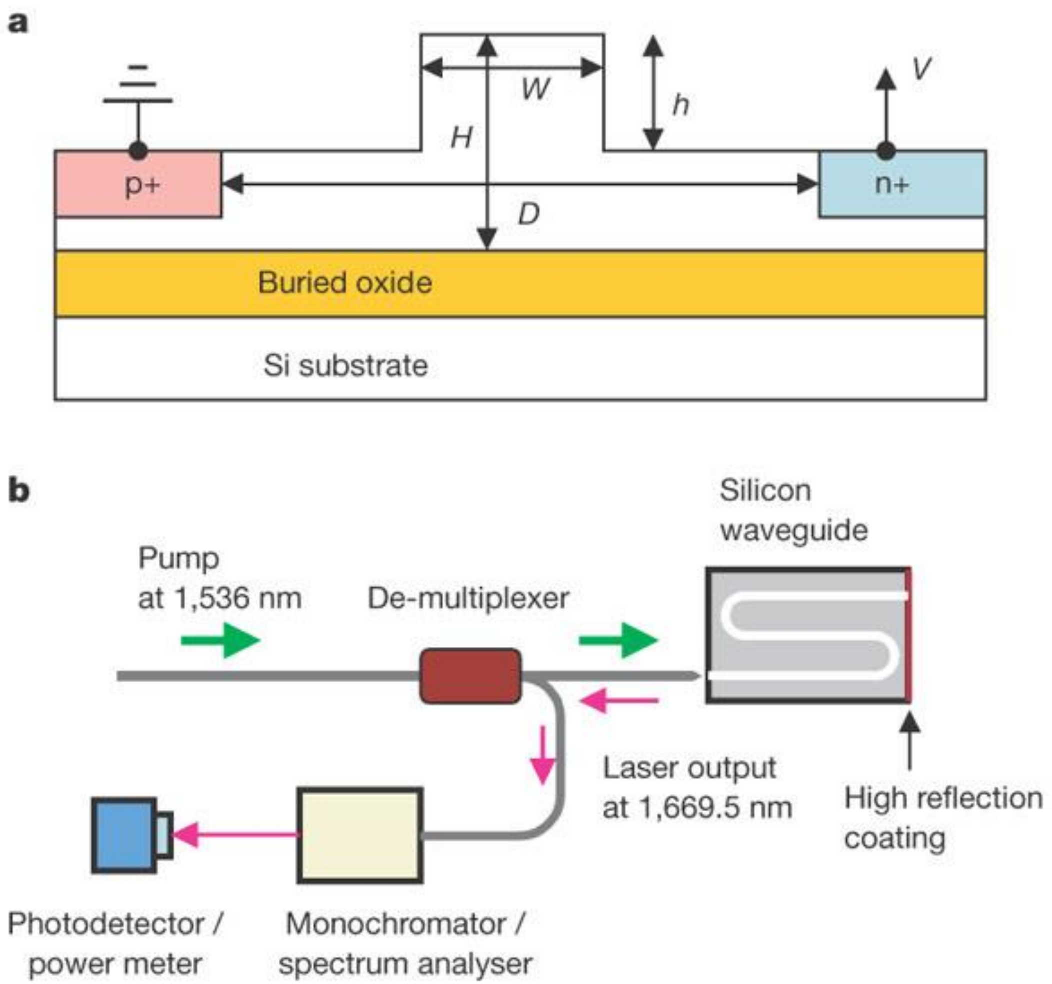


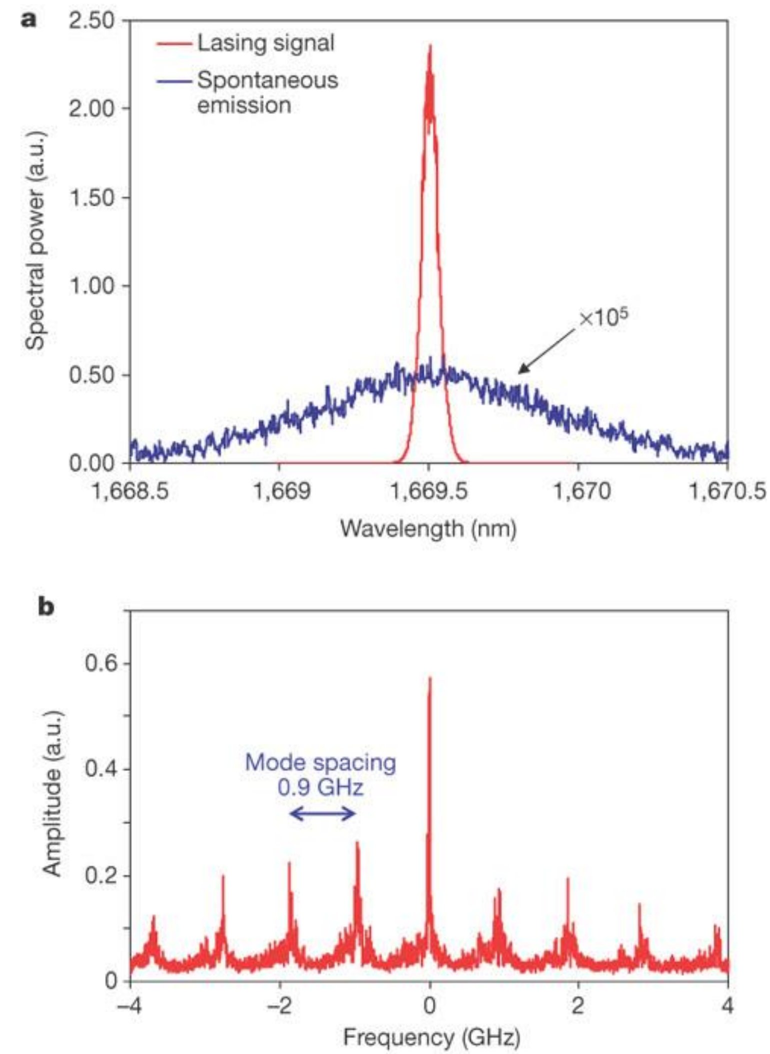

*Manley-Rowe relations*

$$\frac{1}{\omega_s}\frac{dP_s}{dz} = \frac{1}{\omega_i}\frac{dP_i}{dz} = -\frac{1}{\omega_p}\frac{dP_p}{dz}$$

# Stimulated Raman Scattering

Recent results from Intel




H. Rong *et al*, Nature, **433**, 292 (2005)

# Keywords

Travelling-wave amplification
Regenerative amplification
Chirped pulse amplification
Pulse stretching, compression
Nonlinear polarization
Phase-matching
Second harmonic generation
Optical parametric generation and oscillation

# Problems

12.2, 12.3, 12.5, 12.7, 12.8, 12.10
Examples: 12.2, 12.3, 12.4