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The eye and vision were natural topics for optics before the invention of optical instruments and artificial light sources. In fact, the word *optics* has its roots in the Greek noun ὀψις (Latin: *visus*), which means vision. Its adjective is ὀπτικός in the masculine form, and ὀπτική together with the feminine τεχνή, which is art, skill, science. Hence, etymologically, optics means vision science. (In medicine, ophthalmology derives from Greek ὀφθαλμός, the eye. In Latin, the eye is *oculus*).

Vision science may be perceived as a very mature science. However, today visual optics uses and further develops methods like those better known from defense and astronomy, such as wavefront sensing and adaptive optics.

Interestingly, the optical system of the human eye used, e.g., aspheric surfaces and gradient-index optical media, long before these terms were coined, and the science of optics has dealt with the challenges of developing technically aspheric surfaces and gradient index (GRIN) lenses. Complying with the intention of this volume to honor and amuse Adolf Lohmann, I will put some small spotlights on visual optics, with special attention to Christophorus Scheiner who, 380 years ago, described the underlying principle for testing the state of refraction of human eyes, similar to the one used today in the Hartmann-Shack sensor.

Refractive surgery—changing the shape of the cornea through ablation using radiation from an excimer laser—has greatly stimulated scientific and commercial interest in methods to map the topography of the cornea and, more importantly, to measure and analyze the imaging or wavefront transmission properties of the complete optical system of the eye, and to do optical design on the eye to reduce its aberrations (e.g., review Ref. [1]).

Hermann von Helmholtz (1821–1894) is one of the pioneers who extensively investigated the structure of the human eye, including its aberrations, and of the visual system. Unfortunately, what he is most quoted for, second and third hand, is an ironic overstatement made during a popular lecture on the performance of the eye when seen as an instrument. Since there are many different versions of the quotation without reference, I tracked down the source.[2] The quotation, with one additional sentence, is:

> Nun ist es nicht zuviel gesagt, dass ich einem Optiker gegenüber, der mir ein Instrument verkaufen wollte, welches die letztgenannten Fehler hätte, mich vollkommen berechtigt glauben würde, die härtesten Ausdrücke über die Nachlässigkeit seiner Arbeit zu gebrauchen und ihm sein Instrument mit Protest zurückzugeben. In Bezug auf meine Augen werde ich freilich letzteres nicht thun, sondern im Gegentheil froh sein, sie mit ihren Fehlern möglichst lange behalten zu dürfen.*

Through this optical system and by means of his invention Augenspiegel (eye mirror)—in English later termed "ophthalmoscope"—Helmholtz in 1850 was the first to look at a living retina (see Ref. [3]). Almost 150 years later, Liang, Williams, and Miller took noninvasive photographs of the retina with the individual photoreceptors resolved.[4–6] The optical system of the eye is used as part of the fundus camera design; the eye's deficiencies, denounced by Helmholtz, were now compensated for by adding a Hartmann-Shack wave sensor to his ophthalmoscope and correcting the eye's aberrations using adaptive optics.

Among those who paved the way in both aberration theory and clinical practice, we also find Allvar Gullstrand (1862–1930). In the context of cornea reshaping, it is impressive to read Gullstrand's account on how he determined the shape of the cornea by photographing the reflexes of test objects, given the inefficiency of light sources and photographic emulsions before 1900 (Ref. [7]).

Around the same time, M. Tscherning investigated the monochromatic aberrations of the entire optical system of the eye and designed an instrument that he called "Aberroskop."[8] This instrument generated the image of a distant point source of light in front of the retina so the retina perceived a disk. The entrance pupil of the eye was covered by a grid, the projection of which in the patch on the retina was seen as distorted, yielding a kind of spot diagram of the aberrations across the pupil. This principle has recently been combined with an ophthalmoscope into an automatic wavefront-analyzing instrument.[9]

The instrument, which during the last few years has been adopted by visual optics researchers worldwide, is known as the Hartmann-Shack wavefront sensor. It is widely used in astronomy for analyzing wavefront distortion due to atmospheric turbulence for subsequent compensation through adaptive optics (an overview is given in Ref. [10]). In principle, a point source is generated on the retina. The light emerging from this point source passes through the optical system of the eye; in the exit pupil of the eye, i.e., in a plane conjugate to it, the emerging wavefront is sampled by an array of small lenses (Fig. 7.1).

Each lenslet focuses its portion of the wavefront to a spot in its focal plane; the distance between this spot and the optical axis of the corresponding lenslet gives

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[*] An English translation, from Hermann von Helmholtz, *Popular Scientific Lectures on Scientific Subjects*, Vol. 1, p. 194, Routledge/Thoemmes Press, London, 1996 (reprint of the 1895 edition) is as follows:

> Now, it is not too much to say that if an optician wanted to sell me an instrument, which had all these defects, I should think myself quite justified in blaming his carelessness in the strongest terms, and giving him back his instrument. Of couse, I shall not do this with my eyes, and shall be only too glad to keep them as long as I can—defects and all.

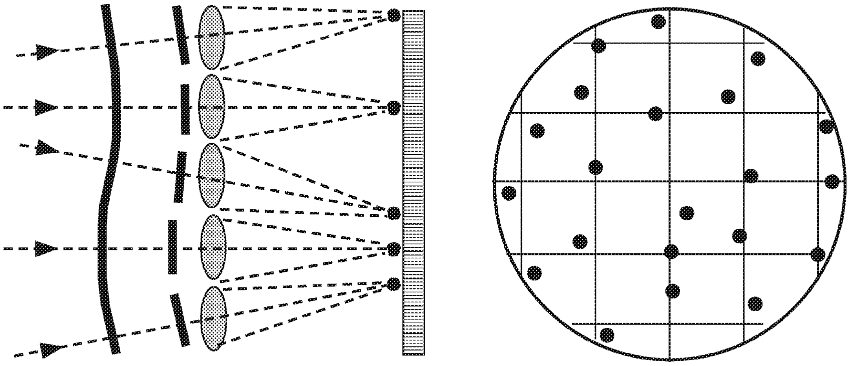


Figure 7.1 Sampling of the local slopes of a wavefront by a Hartmann-Shack sensor (adopted from Thibos[11]).

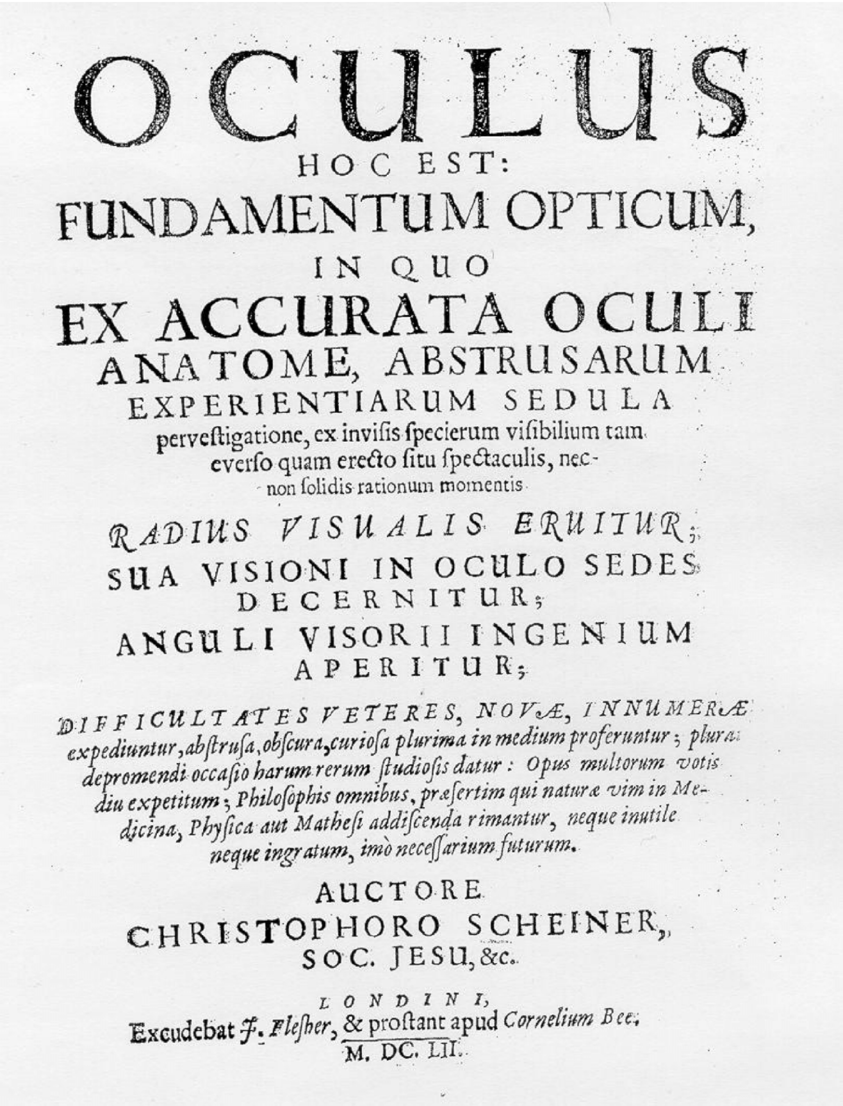
the local slope of the wavefront entering the lenslet. The distribution of spots collected on a CCD matrix allows the calculation of the shape of the whole wavefront, from which the aberration induced by the eye can be described mathematically, e.g., in the form of Zernike polynomials.[12]

To develop a method to test optical instruments, J. Hartmann proposed in 1900 to single out limited bundles of rays going through the optical system and to observe where they pass through planes in front of and behind the focal plane. Hartmann first used this method for the adjustment of spectrographs.[13] He showed that, for measuring the focal length of lenses, the focal plane can be more accurately determined by putting two slits into the entrance pupil and measuring the separation of the two ray bundles in planes on both sides of the focus, rather than by looking for maximum definition where the bundles coincide. Hartmann then developed the method further for testing the aberrations of lenses by placing an array of openings into the entrance pupil of the system under test, as described in detail in an extensive treatise on the testing of optical systems.[14] The Hartmann test became a standard method in optical workshops and laboratories.[15]

The modification of the Hartmann test as shown and explained in Fig. 7.1 was reported by R. V. Shack and B. C. Platt at the 1971 Spring Meeting of the Optical Society of America.[16] The perforated screen was replaced by an array of small lenses arranged downstream of the lens under test.

As to the term commonly used today for the wavefront sensor, Thibos[11] has drawn attention to a precursor to Hartmann, namely the Jesuit Christophorus Scheiner. Reference books report that Scheiner (1575–1650)—a mathematician, physicist, and astronomer—held chairs at the universities of Ingolstadt, Innsbruck, and Freiburg im Breisgau; invented the pantograph in 1603 (for copying drawings at arbitrary scale); and built telescopes. In 1611 Scheiner discovered the sun spots, about which he had long discussions with Galileo; however, being a Jesuit, he could not but advocate the Ptolemaic system. [By the way, Francesco Maria Grimaldi (1618–1663), who described and coined the term *diffraction* (*De lumine* 1665), was also a Jesuit.] In 1619 Scheiner published in Innsbruck a scientific treatise on the human eye, *Oculus, hoc est: Fundamentum opticum* . . . .[17] Figure 7.2 shows the front page of the 1652 edition published in London, with the title going on into a summary of the work.

This is the copy I could inspect at the Royal Library in Stockholm. In 254 pages structured into three books with a total of 113 chapters, Scheiner develops the anatomy of the eye, derives the function of the organs, and deduces step by step that "the retina is the seat of vision." His sectional view of an eye on p. 17, "Sectio oculi per axem et nervum opticum" (a description comprehensible without



OCULUS
HOC EST:
FUNDAMENTUM OPTICUM,
IN QUO
EX ACCURATA OCULI
ANATOME, ABSTRUSARUM
EXPERIENTIARUM SEDULA
perveſtigatione, ex inviſis ſpecierum viſibilium tam
everſo quam erecto ſitu ſpectaculis, nec-
non ſolidis rationum momentis

RADIUS VISUALIS ERUITUR;
SUA VISIONI IN OCULO SEDES
DECERNITUR;
ANGULI VISORII INGENIUM
APERITUR;

DIFFICULTATES VETERES, NOVÆ, INNUMERÆ
expediuntur, abſtruſa, obſcura, curioſa plurima in medium proferuntur; plura
depromendi occaſio harum rerum ſtudioſis datur: Opus multorum votis
diu expetitum; Philoſophis omnibus, præſertim qui naturæ vim in Me-
dicina, Phyſica aut Matheſi addiſcenda rimantur, neque inutile
neque ingratum, imò neceſſarium futurum.

AUCTORE
CHRISTOPHORO SCHEINER,
SOC. JESU, &c.

LONDINI,
Excudebat *J. Fleſher*, & proſtant apud *Cornelium Bee*,
M. DC. LII.

Figure 7.2 Christophorus Scheiner's 1619 treatise on the eye, as described by the front page of the 1652 edition.

translation to everyone in the field because the terminology is still used today), is said to be the first correct presentation of the eye including the optic nerve.

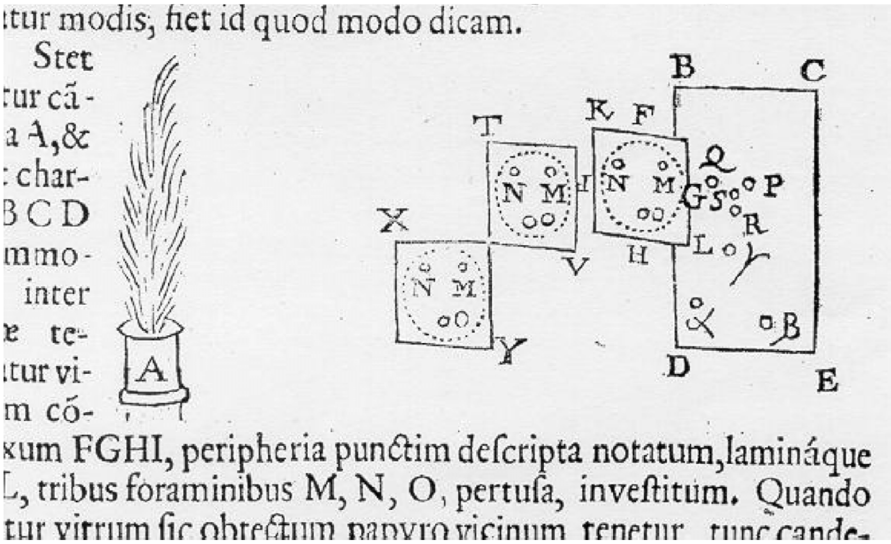
Snell formulated the law of refraction in 1621, but it was not published until 1637 by Descartes. However, Scheiner devotes the whole second book of his treatise to the refraction of light rays through the elements of the eye, surface by surface. In the first part of the third book, on page 164 of the 1652 edition, Scheiner describes the experiment that may be seen as a forerunner to Hartmann: to study the refractive state or focusing, but not aberrations.

Scheiner's description of his raytracing experiment is approximately as follows:

> Have a candle A and a screen BCDE kept in fixed positions, and between them place a convex lens described by the points FGHI around its periphery, which is attached to a board KL into which three holes M, N, O, are pierced. When the lens so covered is held close to the paper, then the candle will be imaged on the screen in triplicate, with the orientation of the candle from each of the three holes upside down; however, the mutual position between the holes is unchanged, if the holes M and N are on top, so also the images P and Q are on the top, and as O is down, so also the corresponding image R is seen at a lower position. The reason is that, as the lens is close to the screen, the images cannot merge in one place, because the distance between the lens and the paper is too short.
>
> When, however, the lens moves towards the candle, gradually the images P, Q, R, converge to one another, so that finally the lens arrives at such a position from which out of all images one image spot S on the screen results. Let this position of the lens be TV, at which only one image and, so to speak, also only one hole (if they are small) are to be seen. From this follows also, that, when the images, thanks to the right refractive state and the convergence of the rays coincide, the image of the candle will keep its reversed orientation, as before.
>
> Now, if the lens from here moves further from the screen and still closer to the candle, not only the single images retain their reversed orientation, but the mutual order from the holes will be interchanged, from top to down, from left to right and right to left. As can be seen, when the lens is at position XY, that the images of candle A, transmitted by the openings M, N, O, will be projected to opposite positions on the paper so that to hole M the corresponding image will be α; N; β; O; γ; . . .

Scheiner transfers these observations to the eye for the study of its refractive state, where the retina has the role of the screen and observes the images of a light source at given distances transmitted through a disk with holes in front of the pupil. Today, this relatively simple test is still known in optometry as "Scheiner disk."[18,19]

Scheiner's method, as explained through Fig. 7.3, lets us understand in an intuitive way that even the laser optometer[20,21] relies on the same principle. A laser optometer consists of a laser-illuminated, slowly rotating, diffusely reflecting cylinder. An observer sees a speckle pattern that barely twinkles when the observer has



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tur vitrum ſic obtectum papyro vicinum tenetur, tunc cande-

Figure 7.3 Scheiner's raytracing experiment with a positive lens drawn in three positions between candle A and screen BCDE. The lens is covered by a board containing three small openings: M, N, and O.

an eye focused close to the cylinder surface; the speckle pattern moves in the same direction as the cylinder surface when the eye is myopic, and in the opposite direction when the eye is hypermetropic.

Herzberger does not mention Scheiner in his overview and bibliography on writers in optics from Euclid to Huygens.[22] However, the name Scheiner may sound familiar due to his namesake, the pioneer of astrophotography Julius Scheiner (1858–1913), whose work on the sensitometry of photographic emulsions resulted in a standard for film sensitivity based on a logarithmic scale in degrees Scheiner (used in Germany until 1934).[23]

But back to eye examination today. Smirnov has been credited with being the first to measure the wave aberrations of the human eye from the lateral aberrations of a thin pencil of light moved sequentially over the entire pupil.[24] The measurements of wave aberrations of the human eye started in Heidelberg[25] in 1994 using a Hartmann-Shack sensor provided by the European Southern Observatory.

The performance of the human eye had been investigated for some years in terms of the modulation transfer function (MTF) through the PSF on the retina by means of a double-pass arrangement. Now, the results from MTF and wavefront measurements are compared by combining both functions in one setup.[26]

When the aberrations of the optical system of an eye are determined, the next step is to correct for them by using adaptive optics. One result, when looking into the eye, is the resolution of single photoreceptor cells of the retina,[4–6] as mentioned at the outset in the paragraph on Helmholtz.

Now that "supernormal vision"[4] is technically feasible, interesting questions arise regarding how much improvement of human vision really is possible.[27,28] We all know, as Helmholtz did, that nature is good at economizing on its resources. The density of neural sampling points in the retina is quite well matched to the resolution in the image projected on them; hence, this density decreases with distance away from the fovea. Our young group at the Royal Institute of Technology in Stockholm, having become involved in the new university education of Swedish optometrists, has chosen to study how people with central scotoma can enhance their peripheral vision—the only vision left to them.[29] The work is done together with the Center for Rehabilitation Research at Lund Institute of Technology. One task is to determine the individual, and by nature large, peripheral aberrations of each scotoma patient's vision (we recently built a Hartmann-Shack sensor). The other task is to map the density of receptors in order to know the extent to which optical corrections can provide recovery.

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